EXTREMAL PROBLEM ON THE B-S BOUNDARY OF A CERTAIN BOUNDED COMPLETE CIRCULAR DOMAIN AND ITS APPLICATIONS

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1. Introduction. As a certain generalization of the Riemann mapping theorem in C, Kubota[4] showed that there exists an extremal mapping $\tilde{f} \in F(D) \equiv Hol^n(D, B_n)$ unique up to unitary linear transformations on C^n, such that, for a fixed $t \in D$,

$$|J_{\tilde{f}}(t)| = \sup\{|J_f(t)| \mid f \in F(D)\},$$

where D, B_n and J_f denote a bounded symmetric domain in C^n, the unit ball $B_n(0,1)$ and the Jacobian determinant $\det(df(z)/dz)$, respectively.

This result is a solution of the Maximalteiler problem due to Carathéodory[1]. We should like to generalize the theorem of Maximalteilers. To go on with this purpose systematically we will start from the minimal problem on the Bergman-Shilov boundary of a certain bounded complete circular domain.

2. Preliminaries. Throughout this paper, let M in C^n be a bounded complete circular domain with starlikeness $t\bar{M} \subset M$ for $t = re^{i\theta}$ $(0 \leqq r < 1)$ and βM denote the Bergman-Shilov boundary of M.

It is known that, for a multiindex $\alpha = (\alpha_1, \cdots, \alpha_n)$ with $|\alpha| = \sum_{i=1}^{n} \alpha_i$, $\{z^\alpha = z_1^{\alpha_1} \cdots z_n^{\alpha_n} \mid z = {}^t(z_1, \cdots, z_n),\ |\alpha| = 0, 1, 2, \cdots\}$ make an orthogonal system on βM and $\dim\{z^\alpha \mid |\alpha| = k\} = {}_{n+k-1}C_k \equiv N(k)$. Therefore we have an orthonormal system $\psi = \{\psi_{kj}\}$ on βM, where ψ_{kj} $(j \equiv j(k) = 1, 2, \cdots, N(k))$ are homogeneous polynomials of degree k $(k = 0, 1, 2, \cdots)$ (see [2]). Further any function $f \in Hol(M)$ has the uniformly convergent series expansion $f(z) = \sum_{k=0}^{\infty} P_k(z)$,

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where each P_k denotes a homogeneous polynomial of degree k. Therefore, put

$$H_2(\beta M)=\{f=\sum_{k=0}^{\infty}\sum_{j=1}^{N(k)} a_{kj}\psi_{kj}\in Hol(M) \mid \sum_{k=0}^{\infty}\sum_{j=0}^{N(k)}|a_{kj}|^2<\infty\},$$

then $H_2(\beta M)$ denotes the $L_2(\beta M, d\sigma_\zeta)$-closure of $A(M)=Hol(M)\cap C(\bar{M})$. The boundary value $f(\zeta)$ $(\zeta\in\beta M)$ is defined by $\lim_{r\to 1-0} f_r(\zeta)$ $(f_r(\zeta)\equiv f(r\zeta),\ 0\leqq r<1)$ which exists a. e. on βM. $H_2(\beta M)$ makes a separable Hilbert space with respect to a base ψ($(\infty\times 1)$type) since for any $f\in H_2(\beta M)$ the Parseval's equality holds.

For two $(n\times 1)$ column vector functions g and h in $L_2(\beta M)$ the inner product $(g,h)_{\beta M}$ is defined by $\int_{\beta M} g(\zeta)h^*(\zeta)d\sigma_\zeta$, and also the scalar product $<g,h>_{\beta M}$ is defined by $\mathrm{Tr}(g,h)_{\beta M}$ with the norm $\|g\|_{\beta M}=<g,g>_{\beta M}{}^{1/2}$, where A^* denotes the transposed conjugate matrix of A.

3. General minimal problem in $H_2(\beta M)$. Let L_t be any bounded linear functional matrix evaluated at $t\in M$, say, $L_t\equiv(1,\partial_z)_{z=t}$ or $\equiv(1,\partial_{z,t})_{z=t}$ etc., where $\partial_z\equiv\partial/\partial z\equiv(\partial/\partial z_1,\cdots,\partial/\partial z_n)$ and $\partial_{z,u}(\cdot)=\partial_z(\cdot)u$ for $z={}^t(z_1,\cdots,z_n)$ and $u={}^t(u_1,\cdots,u_n)\in\mathbf{C}^n-\{0\}$.

Now, we consider a subclass of $H_2{}^n(\beta M)$ as

$$H_2{}^n(\beta M\mid(K,t))=\{f\in H_2{}^n(\beta M)\mid L_t f=K\}$$

for a given constant matrix K of the same type of L_t. Let $m_{(K,t)}{}^G$ denote the minimal $(n\times 1)$ vector function such that $\|G-m_{(K,t)}{}^G\|_{\beta M}=\inf\{\|G-f\|_{\beta M}\mid f\in H_2{}^n(\beta M\mid(K,t))\}$ for a fixed $G\in L_2{}^n(\beta M)$, and $\lambda_{(K,t)}{}^G$ denote the minimal value $\|G-m_{(K,t)}{}^G\|_{\beta M}{}^2$.

THEOREM 1. *Put* $L_t\psi=\Psi$ *and* $B=(G,\psi)_{\beta M}$($(n\times\infty)$ *type*) *for fixed* K *and* $G\in L_2{}^n(\beta M)$, *then we have*

(3.1) $$m_{(K,t)}{}^G(z)=\{B+(K-B\Psi)(\Psi^*\Psi)^{-1}\Psi^*\}\psi(z)\in H_2{}^n(\beta M\mid(K,t))$$

and

(3.2) $$\lambda_{(K,t)}{}^G=\|G\|_{\beta M}{}^2-\mathrm{Tr}\{BB^*-(K-B\Psi)(\Psi\Psi^*)^{-1}(K-B\Psi)^*\},$$

where $\Psi^*\Psi$ *is positive definite.*

Proof. For a sufficiently large number L, the restricted class

$\{f\in H_2{}^n(\beta M\mid(K,t))\mid\|G-f\|_{\beta M}\leq L\}$ makes a compact family. Then there exists the extremal mapping $m\in H_2{}^n(\beta M\mid(K,t))$ such that $\|G-m\|_{\beta M}=\inf\{\|G-f\|_{\beta M}\mid f\in H_2{}^n(\beta M\mid(K,t))\}$. Noting that m has its Fourier expansion $A\psi$ with the $(n\times\infty)$ matrix coefficient $A=(a_{ij})=(m,\psi)_{\beta M}$, put

$$J(A)=\|G-m\|_{\beta M}{}^2-\mathrm{Tr}\{(A\Psi-K)\Lambda+\Gamma^*(A\Psi-K)^*\}$$

with the Lagrangian multiplier matrices Λ and Γ of the same type of ${}^tK$. Noting that $\|G-m\|_{\beta M}{}^2=\|G\|_{\beta M}{}^2-\mathrm{Tr}\{AB^*+BA^*-AA^*\}$, from the Euler's conditions $\partial J(A)/\partial a_{ij}=0$ and $\partial J(A)/\partial\bar{a}_{ij}=0$ we have $A-B=(\Psi\Lambda)^*=(\Psi\Gamma)^*$. Since $K=L_t m=A\Psi=(B+(\Psi\Lambda)^*)\Psi$, we must have $\Lambda^*=\Gamma^*=(K-B\Psi)(\Psi^*\Psi)^{-1}$ with a positive definite Hermitian matrix $\Psi^*\Psi$ from the existence of the extremal function, that is, $m=\{B+(K-B\Psi)(\Psi^*\Psi)^{-1}\Psi^*\}\psi\in H_2{}^n(\beta M\mid(K,t))$.

In order to confirm that m gives the minimized mapping just required, put $\tilde{f}=f-m$ for any $f\in H_2{}^n(\beta N\mid(K,t))$ and satisfies $\mathrm{Tr}(G-m,\tilde{f})_{\beta M}=0$. Therefore, we get $\|G-f\|_{\beta M}{}^n=\|G-m\|_{\beta M}{}^2+\|\tilde{f}\|_{\beta M}{}^2\geq\|G-m\|_{\beta M}{}^2$, where the equality holds only for $\|\tilde{f}\|_{\beta M}=0$, i.e., $\tilde{f}(z)\equiv 0$. This shows that $m=m_{(K,t)}{}^G$. Finally, we have the minimal value $\lambda_{(K,t)}{}^G$ by direct calculations.

COROLLARY. *Put $m_{(K,t)}{}^G|_{G=0}=m_{(K,t)}$ and $\|m_{(K,t)}\|_{\beta M}{}^2=\lambda_{(K,t)}$, then we have*

(3.3) $$m_{(K,t)}(z)=K(\Psi^*\Psi)^{-1}\Psi^*\psi(z)\in H_2{}^n(\beta M\mid(K,t)),$$

(3.4) $$\lambda_{(K,t)}=\mathrm{Tr}(K(\Psi^*\Psi)^{-1}K^*).$$

LEMMA 1. (1) *Put $S_M(z,t)=\psi^*(t)\psi(z)$, then we have*

(3.5) $$0<S_M(z,z)<\infty,\quad z\in M,$$

(3.6) $$S_M(\cdot,z)\in H_2(\beta M)\ \text{with}\ S_M(\cdot,z)=S_M(z,\cdot)^*,\quad z\in M,$$

(3.7) $$f(z)=(f(\cdot),S_M(\cdot,z))_{\beta M},\quad f\in H_2(\beta M),\quad z\in M,$$

(3.8) $$S_M(z,0)=S_M(0,0)=1/\mathrm{vol}(\beta M),\quad z\in M.$$

(2) *Put $H_M(z,t)=\partial_z{}^*\partial_z\log S_M(z,t)=\{(S_M\times\partial_z{}^*\partial_z S_M-\partial_z{}^*S_M\times\partial_z S_M)/S_M{}^2\}(z,t)$, then we have*

(3.9) $$H_M(z,0)=H_M(0,0)=\mathrm{vol}(\beta M)\,\partial_z{}^*\partial_z S_M(0,0)=\mathrm{vol}(\beta M)B_1{}^*B_1,\quad z\in M,$$

(3.10) $$H_M(z,z)\ \textit{is positive definite for}\ z\in M,$$

where $B_1 z$ ($(n\times 1)$ type) denotes the linear part of ψ.

Proof. (1) Let $K=1=f(t)=L_t f$, then from (3.4) we have $0<\lambda_{(1,t)}=1/S_M(t,t)<\infty$, which gives (3.5). On the other hand, $m_{(1,t)}=S_M(z,t)/S_M(t,t)$ belongs to $H_2(\beta M|(1,t))$, which gives $S_M(\cdot,t)\in H_2(\beta M)$. $S_M(\cdot,z)=S_M^*(z,\cdot)$ is obvious. For any $f=\sum_{i,j} a^f{}_{ij}\psi_{ij}\in H_2(\beta M)$ we have $(f(\ ),S_M(\ ,z))_{\beta M}=f(z)$ from $(\psi_{kj},\psi_{k'j'})_{\beta M}=\delta_{(kj),(k'j')}=1$ for $(k,j)=(k',j')$ and $=0$ elsewhere. Since $S_M(z,0)=S_M(0,0)$ is clear from the definition of S_M, then $S_M(0,0)=(S_M(\ ,0),S_M(\ ,0))_{\beta M}=S_M{}^2(0,0)\mathrm{vol}(\beta M)$ from (3.7), which shows (3.8).

(2) By the definition of H_M, (3.8) and $\partial_z{}^*\partial_z S_M(0,0)=B_1{}^*B_1$, we have (3.9). For $K=(0,1)=L_t f=(1,\partial_{z,u})_{z=t}f$, we have, from (3.4), $0<\lambda_{(K,t)}=(S_M(t,t)\cdot u^*H_M(t,t)u)^{-1}<\infty$ for all $u\in\mathbf{C}^n-\{0\}$, which gives (3.10).

REMARK 1. $S_M(z,t)$ and $H_M(z,t)$ are called the Szegö kernel and the Szegö tensor on M, respectively. The Szegö projection $(\mathrm{P}f)(z)=(G(\ ),S_M(\ ,z))_{\beta M}$ for $G\in L_2(\beta M)$ is a Hilbert space projection of $L_2(\beta M)$ onto $H_2(\beta M)$. In particular $(\mathrm{P}f)(z)=f(z)$ holds for $f\in H_2(\beta M)$(see (3.7)), which is called the reproducing property of S_M on $H_2(\beta M)$.

LEMMA 2. *Let $H_2{}^n(\beta M|\tilde{K})$ denote the normalized subclass of $H_2{}^n(\beta M)$ with $K=(P,Q)=L_0 f=(1,\partial_z)_{z=0}f$, where $P\in\mathbf{C}^n$ and $|\det Q|=1$. $m_{\tilde{K}}$ and $\lambda_{\tilde{K}}$ denote an $H_2{}^n(\beta M|\tilde{K})$-minimal mapping and the $H_2{}^n(\beta M|\tilde{K})$-minimal value $\|m_{\tilde{K}}\|_{\beta M}{}^2$, respectively. Then we have*

$$(3.11)\qquad m_{\tilde{K}}(z)=Q_0 z,\quad Q_0=U(\det H_M(0,0))^{-1/2n}H_M{}^{1/2}(0,0),\quad U^*U=E_n,$$

where E_n denotes the unit matrix of order n, and

$$(3.12)\qquad \lambda_{\tilde{K}}=n\,\mathrm{vol}(\beta M)(\det H_M(0,0))^{-1/n},$$

where $H_M{}^{1/2}$ denotes $V_0{}^*\mathrm{diag}(\mu_1,\cdots,\mu_n)V_0$ for $H_M(0,0)=V_0{}^*\mathrm{diag}(\mu_1{}^2,\cdots,\mu_n{}^2)V_0$ with $\mu_1\geqq\cdots\geqq\mu_n>0$ and $V_0{}^*V_0=E_n$.

Proof. From Corollary for $K=(P,Q)=L_0 f=(1,\partial_z)_{z=0}f$, we have

$$(3.13)\qquad m_{(K,0)}(z)=P+Q(\partial_z{}^*\partial_z S_M(0,0))^{-1}\partial_z{}^*S_M(z,0)=P+Qz$$

since $\partial_z m_{(K,0)}(z)=Q$ holds from $\partial_z{}^*\partial_z S_M(z,0)=\partial_z{}^*\partial_z S_M(0,0)=B_1{}^*B_1$, and also we have, from (3.8),

(3.14) $\lambda_{(K,0)}=\mathrm{vol}(\beta M)(|P|^2+\mathrm{Tr}\{QH_M^{-1}(0,0)Q^*\}\geq \mathrm{vol}(\beta M)\mathrm{Tr}\{QH_M^{-1}(0,0)Q^*\}$.

For all $P\in C^n$ and Q with $|\det Q|=1$, by making use of the diagonal representation $QH_M^{-1}(0,0)Q^*=V^*\mathrm{diag}(\lambda_1,\cdots,\lambda_n)V$, $V^*V=E_n$, we have

$$\mathrm{Tr}\{QH_M^{-1}(0,0)Q^*\}=\sum_{i=1}^n\lambda_i\geqq n(\prod_{i=1}^n\lambda_i)^{1/n}=n(\det H_M(0,0))^{-1/n},$$

where the equality holds only for $\lambda_1=\cdots=\lambda_n=\lambda=(\det H_M(0,0))^{-1/n}$, i.e., $Q_0H_M^{-1}(0,0)Q_0^*=\lambda E_n$ with $Q_0=U\lambda_{1/2}H_M^{1/2}(0,0)=U(\det H_M(0,0))^{-1/2n}H_M^{1/2}(0,0)$, $U^*U=E_n$. Hence we obtain (3.12) and (3.11) from (3.14) and (3.13), respectively, since $\lambda_{(K,0)}\geqq \mathrm{vol}(\beta M)\mathrm{Tr}\{Q_0H_M^{-1}(0,0)Q_0^*\}=n\mathrm{vol}(\beta M)\lambda=\lambda_{\tilde K}$.

4. β-canonical domains. The image domain $\tilde M$ of M under $m_{\tilde K}$ is called an $H_2^n(\beta M|\tilde K)$-minimal domain and $m_{\tilde K}$ is called an $H_2^n(\beta M|\tilde K)$-minimal mapping.

THEOREM 2. *For M with Szegö kernel S_M, the following statements are equivalent each other.*

(4.1) *M itself is an $H_2^n(\beta M|\tilde K)$-minimal domain.*

(4.2) *M has* Prop(B): $H_M(0,0)=b^2E_n$, $b>0$.

(4.3) *M has* Prop(B'): $(\zeta,\zeta)_{\beta M}=b'^2E_n$, $b'>0$ [4].

Let Prop(β) *be the general term for above conditions, then M with* Prop(β) *(or a β-canonical domain M) satisfies*

(4.4) $$m_{\tilde K}(z)=Uz,\quad U^*U=E_n,$$

(4.5) $$\lambda_{\tilde K}=n\mathrm{vol}(\beta M)/b^2=nb'^2.$$

Proof. (4.1) is equivalent to $Q_0z\equiv z$, i.e., $Q_0=E_n$, which gives (4.2) from (3.11) and vice versa. Let B_1z be the linear part of ψ, then we have $E_n=(B_1\zeta,B_1\zeta)_{\beta M}=B_1(\zeta,\zeta)B_1^*$, i.e., $(\zeta,\zeta)_{\beta M}=(B_1^*B_1)^{-1}=\mathrm{vol}(\beta M)H_M^{-1}(0,0)=\mathrm{vol}(\beta M)b^{-2}=b'^2E_n$ from (4.2) and (3.9). This gives (4.2)$\Leftrightarrow$(4.3). (4.4) and (4.5) are given by Lemma 2.

LEMMA 3. *Let M be a β-canonical domain with the minimal value $\lambda_{\tilde K}(=\|m_{\tilde K}\|_{\beta M}^2)$, then we have*

(4.6) $$|J_f(0)|^2 \leqq 1,\quad f \in H_2{}^n(\beta M \mid \|f\|_{\beta M}{}^2 \leqq \lambda_{\bar{K}}),$$
where the equality holds only for $f(z)=Uz$, $U^*U=E_n$.

Proof. For any $f \in H_2{}^n(\beta M \mid \|f\|_{\beta M}{}^2 \leqq \lambda_{\bar{K}})$ with $K=(P,Q)=L_0 f=(1,\partial_z)_{z=0} f$, we have, from (3.14) and (4.2),
$$\lambda_{\bar{K}} \geqq \|f\|_{\beta M}{}^2 \geqq \lambda_{(K,0)} \geqq \lambda_{((0,Q),0)} = \mathrm{vol}(\beta M)\,\mathrm{Tr}(QQ^*)/b^2 \geqq \lambda_{\bar{K}} |J_f(0)|^{2/n},$$
which gives (4.6). The equality of (4.6) holds only for $|J_f(0)|=1$ and consequently $\|f\|_{\beta M}{}^2=\lambda_{\bar{K}}$. Therefore from (4.4) we obtain $f(z)=Uz$, $U^*U=E_n$.

DEFINITION. If a bounded domain D in C^n satisfies

(4.7) $$\beta D \subset \partial B_n(0,\rho_D),\quad \rho_D=\inf\{\rho \mid D \subset B_n(0,\rho)\},$$
it is said that D has Prop(∂) with respect to 0.(see Remark 2 (1)).

THEOREM 3. *Let* M *be a* β*-canonical domain with* Prop(∂), *then we have*

(4.8) $$|J_f(0)| \leqq 1,\quad f \in Hol^n(M, B_n(0,\rho_M)).$$

If g *belongs to* $\{f \in Hol^n(M) \mid |J_f(0)| \geqq 1\}$, *then we have*

(4.9) $$\rho_M \leqq \rho_{g(M)}.$$

The equalities in (4.8) *and* (4.9) *hold simultanaously only for unitary linear transformations.*

Proof. For $f \in Hol^n(M, B_n(o,\rho_M))$ we have, from (4.3) and (4.5), $\|f\|_{\beta M}{}^2 \leqq \int_{\beta M} \rho_M{}^2 d\sigma_\zeta = \mathrm{Tr}(\zeta,\zeta)_{\beta M} = nb'^2 = \lambda_{\bar{K}}$, which gives (4.8) from (4.6), where the equality holds only for $f(z)=Uz$, $U^*U=E_n$.

Suppose that g belongs to $Hol^n(M \mid |J_f(0)| \geqq 1,\ f(z) \neq Uz, U^*U=E_n)$, then $g(M)-B_n(0,\rho_M) \neq \emptyset$ (g does not belong to $Hol^n(M, B_n(0,\rho_M))$), that is, $\rho_M < \rho_{g(M)}$. $\rho_M=\rho_{g(M)}$ holds only for $g(z)=Uz$, $U^*U=E_n$.

REMARK 2. (1) Any starlike and homogeneous bounded complete circular domain N is equivalent to a β-canonical domain M with Prop(∂) under some linear mapping (see Lemma 2, (4.4) and [2,4]).

(2) (4.9) in Theorem 3 gives a geometrically simple characterization such that a β-canonical domain M with Prop(∂) is the most condenced domain in the $\{f \in Hol^n(M) \mid |J_f(0)|=1\}$-equivalent class of domains.

(3) Any bounded symmetric domain Y, say, a classical Cartan domain, is a homogeneous convex complete circular domain. The explicit formulas of the Szegö kernel $S_Y(z,t)$ and the Bergman kernel $K_Y(z,t)$ are given in [3]. A β-canonical domain $\tilde{Y}$ of Y has also Prop(∂)(see (1) above).

5. Maximalteilers. For two bounded domain D *and* D' in $\mathbf{C}^n$ with $0 \in D \cap D'$, we consider a class $H_0=\{f \in Hol^n(D,D') \mid f(0)=0\}$. If $\tilde{f}$ is an extremalmapping which satisfies $|J_{\tilde{f}}(0)|=\sup\{|J_f(0)| \ f \in H_0\}$, then $\tilde{f}(D)$ is a Maximalteiler in D'. Now, we should like to generalize Theorem 15 given by Carathéodory[1] on the Maximalteilers for $Hol^n(P_n, B_n(0,1))$, where P_n denotes a polycylinder.

The following Theorem 4 is immediately obtained by Theorem 3.

THEOREM 4. *Let M be a β-canonical domain with* Prop(∂), *then M is a unique Maximalteiler in a ball $B_n(0,\rho_M)$ up to unitary linear transformations*. (cf. [1] Theorem 15).

THEOREM 5. *Let D be a biholomorphic image of M with homogeneity, then there exists an extremal mapping $\tilde{f}$ in $F(D)=\{f \in Hol^n(D,B_n(0,1) \mid f(t)=0,\ t \in D\}$, unique up to unitary linear transformations in* $\mathbf{C}^n$, *such that*

(4.10) $$|J_{\tilde{f}}(t)|^2=\sup\{|J_f(t)| \mid f \in F(D)\}=\det T_D(t,t)/\det T_{\tilde{M}}(0,0),$$

where $\tilde{M}=\tilde{f}(D)$ $(\subset B_n(0,1))$ is a β-canonical domain with Prop(∂) *and also $\tilde{f}$ is given by the curvilinear integral such that*

(4.11) $$\tilde{f}(z)=UT_{\tilde{M}}^{-1/2}(0,0)U_0T_D^{-1/2}(t,t)\int_t^z T_D(z,t)\,dz,\quad U^*U=E_n,$$

where $T_D(z,t)=\partial_{z^*}\partial_z \log K_D(z,t)$ denotes the Bergman tensor with respect to the bergman kernel $K_D(z,t)$ of D and U(resp. U_0) is an *arbitrary(resp. a certain) unitary matrix.*

Proof. There exists a biholomorphic mapping g of D onto M with $g(t)=0$ for any t in D. Since D is mapped onto any Maximalteiler M' of $B_n(0,\rho_{M'})$ by $UQ_0\circ g$ (Q_0 in (3.11) and $U^*U=E_n$), then $\tilde{f}=\rho_{M'}{}^{-1}UQ_0\circ g$ maps D onto any Maximalteiler $\tilde{M}$ of $B_n(0,1)$ with $\tilde{f}(t)=0$. Noting that $\eta=\rho_{M'}{}^{-1}Q_0\circ g$ is a biholomorphic mapping of D onto a Maximalteiler $\tilde{M}$ of $B_n(0,1)$, we have

(4.12) $\quad T_D(z,t)=(\partial_z\eta(t))^*T_{\tilde{M}}(\eta(z),\eta(t))\partial_z\eta(z)=(\partial_z\eta(t))^*T_{\tilde{M}}(0,0)\partial_z\eta(z)$

from the biholomorphic relative invariance of the Bergman tensor $T_D(z,t)$ and $T_{\tilde{M}}(\eta(z),0)\equiv T_{\tilde{M}}(0,0)$ for a complete circular domain $\tilde{M}$. Hence we have (4.10) from $|J_{\tilde{f}}(t)|^2=|J_{U\eta}(t)|^2=|J_U(0)|^2|J_\eta(t)|^2=|J_\eta(t)|^2\equiv|\det\partial_z\eta(z)|^2$ and (4.12). Further, we have $\partial_z\eta(z)=T_{\tilde{M}}^{-1}(0,0)(\partial_z\eta(t))^{*-1}T_D(z,t)=$ $=T_{\tilde{M}}^{-1/2}(0,0)U_0T_D^{-1/2}(t,t)T_D(z,t)$ from (4.12). Therefore, we get (4.11) from the curvilinear integral $\tilde{f}(z)=U\eta(z)==U\int_t^z\partial_z\eta(z)dz$ with $\eta(t)=0$.

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(Received November 17, 1995)