Possibility of Protonium Molecules

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Possibility of having protonium molecules is speculated in a language parallel with that of postitronium molecule formation. The binding energy of the suggested protonium molecule is expected to be 750 eV.

In the course of proton-antiproton ($p-\bar{p}$) interaction studies, existence of antiprotonic hydrogen (protonium) atoms has come into limelight [1]. The protonium atom can be built up by replacing the electron in a hydrogen atom by antiproton. It is a hadron version of a positronium atom, in which a positron and an electron constitute a hydrogen atom-like particle pair. In considering proton and antiproton coupling, one has to take the strong interaction feature into account in parallel with the Coulomb interaction. The Coulomb interaction, however, outweighs the other far more at a distance of the protonium Bohr radius, or $a_0(p\bar{p}) = 57.6$ fm, by a factor of more than 10^{13}. Accordingly, a proton-antiproton pair can be expected to form a metastable hydrogen atom-like structure for a reasonable time. As a matter of fact, even absolute X-ray intensities for Lyman and Balmer series have been measured. One can thus consider the existence of protonium atoms as an established reality.

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The purpose of this note is to suggest a possibility of protonium molecule formation in a way similar to the positronium molecule concept from positronium atoms. The concept of positronium molecules dates back to 1947, when Hylleraas and Ore calculated the binding energy to be 0.116 eV [2]. In the absence of experimental studies, theoretical calculation for the same system has been extended [3-7]. One of the newest variational values for the binding energy is by Ho [6], yielding 0.0302 Ry, that is in good agreement with the Green's function Monte Carlo method by Lee *et al.* [7], which gives 0.0303 Ry. Giving credit to these recent calculations, one may assume a tentative value of 0.03 Ry for the binding energy of positronium molecules.

Paraphrasing for the case of protonium molecules is straightforward. All we have to do is replacing the positronium Rydberg with the protonium Rydberg as well as replacing the positronium Bohr radius, or twice the proper hydrogenic Bohr radius with the $p-\bar{p}$ distance at the $1S$ state of protonium atoms.

The energy levels of protonium atoms can be written with conventional notations:

$$E_n = -(1/n^2)Ry_{p\bar{p}}$$

with

$$Ry_{p\bar{p}} = (1/2)\mu c^2\alpha^2$$
$$= 12.5 \text{ keV}$$

and

$$\mu = (1/2)m_p$$

Introducing the above-mentioned binding energy of positronium molecules with

replacement of the Rydberg constant, one finds the binding energy of protonium molecules to be 750 eV. This value is considered large enough to sustain metastability of the molecule.

If a sufficiently dense $p-\bar{p}$ gas could be produced, accordingly, a certain amount of protonium molecules should be born. This is in analogy with formation of hydrogen molecules out of a nascent hydrogen gas. Since the protonium formation is expected to start at relatively large principal quantum numbers ($n \sim 30$) [8], the onset of molecule formation would follow the suit and may occur at reasonably large $p \cdot \bar{p}$ distances.

A protonium molecule consists of four particles of equal mass. Interaction between particles, of two plus charges and two minus charges, satisfies a kind of symmetry condition, yielding a triangular pyramid of the same edge length of $5.9(a_0(p\bar{p})/2) = 170$ fm, in accordance with Ho's variational calculation for positronium molecules [6]. Such a symmetric form as a triangular pyramid enhances a structural stability of the exotic molecule all the more, even in the presence of high probability for matter-antimatter annihilation. As a matter of fact, the $p-\bar{p}$ distance of 170 fm is considerably larger than that of a protonium atom. A protonium molecule, accordingly, once formed, may have a reasonably long lifetime to cope with that of a protonium atom.

In the practical absence around of thick matter-antimatter plasma, steady formation of protonium molecules appears to be very unlikely. Such a scaling calculation of binding energy as presented here, might then be next to nonsense, or at least too academic. A similar argument, however, could also

have been raised against positronium molecules. The concept of positronium molecules indeed survived a long term of little attention until the discovery of excitonic molecules in semiconductors. As is well known, the concept of positronium has been transplanted into the solid state physics in the name of exciton, an electron-hole complex, with hole playing the role of positron in the case of positronium. Then emerges the idea of excitonic molecules. A variational calculation has yielded binding energy of the molecule as a function of hole to electron, or electron to hole, mass ratio [4]. The minimum binding energy is given as the mass ratio becomes unity, corresponding to the case of positronium molecules, while the maximum occurs at the infinite mass ratio, or practically corresponding to the case of hydrogen molecules. In short, unbalance in mass increases the binding energy. A further important feature of the excitonic system in solids, particularly in a typical semiconductor material, like silicon or germanium, is a stable association with a neutral impurity. In general, the binding energy of an exciton with a neutral impurity -----either donor or acceptor-----is several times larger than that of an excitonic molecule. Back to the present case of the heavier particle pair once more, one may expect that a protonium atom can also be more strongly bound to a foreign atom, practically of any kind, than to another protonium atom.

Thus a wide variety of physics concerning transient exotic atoms and molecules, involving not only baryons but also leptons or mesons possibly, can be considered ready for use in new interpretation of local or nonlocal evolutions in the nature. The protonium molecule formation indicates just a beginning of the variety.

REFERENCES

1) To have a summarizing knowledge, see, for example, G. Reifenrother and E. Klempt: Nucl. Phys. A503 (1989) 885.

2) E. A. Hylleraas and A. Ore: Phys. Rev. 71 (1947) 493.

3) A. Ore: Phys. Rev. 71 (1947) 913.

4) O. Akimoto and E. Hanamura: Solid State Commun. 10 (1972) 253.

5) W. F. Brinkman, T. M. Rice, and B. Bell: Phys. Rev. 58 (1973) 1570.

6) Y. K. Ho: Phys. Rev. A33 (1986) 3584.

7) M. A. Lee, P. Vashishta, and R. K. Kalia: Phys. Rev. Lett. 51 (1983) 2422.

8) J. A. Cohen: Phys. Rev. A36 (1987) 2024.

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