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MODELING OF MAGNETIC LEVITATION OF HIGH TC SUPERCONDUCTOR BY ENERGY CONVERSION

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ABSTRACT

Bulk HTSC(YBCO) and a permanent magnet both of cylindrical shape are placed vertically in a coaxial line. When the magnet is approached from the lower side to the HTSC placed upper side, the current is induced in the HTSC to levitate itself.

While the magnet is departed, the current is induced in the reverse direction to attract the magnet so called fishing. We propose the physical essence that the mechanical energy to move the magnet is converted to the electromagnetic energy by the velocity emf. We analyze these phenomena by replacing the bulk HTSC by the coil without resistance.

1. INTRODUCTION

We assume the configuration shown in Figure1. When the magnet is approached from the lower side, the current is induced in the HTSC to levitate itself. While the magnet is departed, the current is induced to the reverse direction to attract the magnet.

The mechanical energy to move the magnet is converted to the electromagnetic energy via the velocity electromotive force (emf). Representing this configuration by the electric circuit composed of coils with mutual inductance, we analyze the interactive force which varies with the gap length between the HTSC and the magnet.

2. MODELING OF LEVITATION BY ELECTROMECHANICAL ENERGY CONVERSION

A bulk HTSC showen in Figure1 is assumed as one turn coil with perfect conduction, although the bulk SC can be represented by multiturn coils with superconducting critical current density[1]. In order to grasp the physical essence of levitation and fishing, we adopt this approximation by choosing the adequate circuit parameters . A uniformly magnetized permanent magnet can be represented by one turn coil with constant current. Figure2(a) and (b) illustrate the scheme for approach and depart, respectively.

<u>Principle of levitation and fishing based on velocity electromotive force</u>

For the approach of magnet: While the HTSC which is replaced by the coil with current I is kept stationary, the coil cuts the radial component of magnetic fields of the permanent magnet $\vec{B}$ which are oriented outward downward by $d\vec{z}$ with velocity $\vec{v}$, accompa -

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nied with the upward movement of magnet. It will generate velocity electromotive force $\vec{v}\times\vec{B}$ increase the magnetic flux repulsive to the magnet. The mechanical work dW_m done for the movement:

$$dW_m=\oint\left(Id\vec{l}\times\vec{B}\right)\bullet d\vec{z}=\oint\left(\vec{B}\times d\vec{z}\right)\bullet Id\vec{l}=\oint\left(\vec{B}\times\vec{v}\,dt\right)\bullet Id\vec{l}=\oint\left(d\vec{z}\times d\vec{l}\right)\bullet I\vec{B}, \quad (1)$$

$$d\vec{z}=\vec{v}dt$$

where, the area segment $d\vec{s}=d\vec{z}\times d\vec{l}$ is oriented inward opposite to the radial component of $\vec{B}$, hence dW_m becomes minus, which means the mechanical work is done to change the magnetic energy.

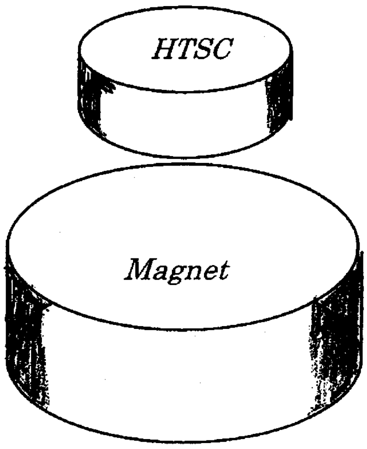
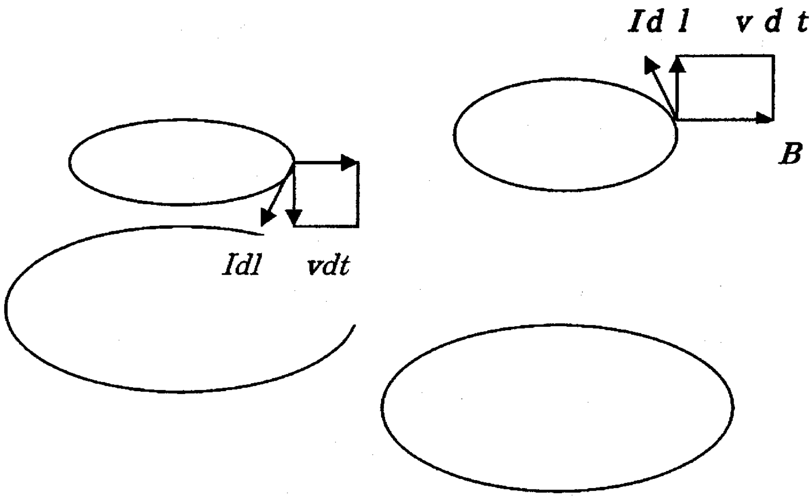


Figure1. HTSC and permanent Magnet.



(a) approach (b) departure

Figure 2. Principle of electromechanical energy Conversion.

We note the force applied to the superconducting one turn coil by the magnetic fields $\vec{F}=\oint Id\vec{l}\times\vec{B}$ is oriented upward, and also $\vec{v}\times\vec{B}$ is oriented to increase the coil current repulsive to the magnet. The change of magnetic energy dW_e is also related to the change of flux linkage as [2] :

$$dW_e=\oint\left(\vec{E}I\bullet d\vec{l}\,dt\right)=\oint\vec{E}\bullet d\vec{l}\,Idt=-\left(\frac{d}{dt}\int_s\vec{B}\bullet d\vec{s}\right)Idt=-\left\{\oint\frac{\partial\vec{B}}{\partial t}+\left(\vec{B}\times\vec{v}\right)\bullet d\vec{l}\right\}Idt \quad (2)$$

$$=-\oint\left(\vec{B}\times\vec{v}\right)\bullet d\vec{l}\,Idt$$

The equation from (2) shows the change of flux linkage of the moving coil is equal to the flux cut by the HTSC coil and the mechanical input energy is equal and opposite to the change of magnetic energy,

which means the mechanical energy done by the external force to close the gap is converted to the electrical energy

$$dW_e = -dW_m \tag{3}$$

For the departure of magnet: The one turn coil cuts upward the radial components of magnetic fields which are also oriented outward, which will generate velocity electromotive force to decrease the repellant current to zero. Finally the current is increased to attract the magnet, so called fishing.

Formulation by lumped electrical circuits

We shall adopt a lumped circuit model composed of the two coils with the mutual inductance which varies with the distance between the coils. We assume for the magnet with large magnetization, the change of magnetic flux linkage can be neglected. Then the induced voltage becomes zero:

$$0 = L_1 \frac{dI_1}{dz} + \frac{dM(z)}{dz} I_1 + M(z)\frac{dI_2}{dz} = \frac{dM(z)}{dz} I_1 + M(z)\frac{dI_2}{dz},$$

$$\because I_1 = I_0 = const., \quad \frac{dI_1}{dz} = 0$$

The superconducting coil induced voltage is short circuited and becomes zero due to superconductivity:

$$0 = L_2 \frac{dI_2}{dz} + M(z)\frac{dI_1}{dz} + \frac{dM(z)}{dz} I_2 = L_2 \frac{dI_2}{dz} + \frac{dM(z)}{dz} I_2,$$

Considering the transformer emf.'s are not included in the two equations, add them side by side:

$$0 = \{L_2 + M(z)\}\frac{dI_2}{dz} + \frac{dM(z)}{dz} I_1 + \frac{dM(z)}{dz} I_2 \tag{4}$$

where, z : distance between the coil and magnet, I_1, I_2 : currents for the magnet and coil, respectively, L_2 : self inductance of coil, M : mutual inductance between the coil of HTSC and magnet.

(4) becomes integrative form as:

$$\frac{dI_2 / dz}{I_1 + I_2} = -\frac{dM(z)/dz}{L_2 + M(z)}$$

$$[\ln(I_1 + I_2)]_{z1}^{z2} = -[\ln(L_2 + M(z))]_{z1}^{z2} \quad \ln\left[\frac{I_1(z_2) + I_2(z_2)}{I_1(z_2) + I_2(z_2)}\right] = -\ln\left[\frac{L_2 + M(z_2)}{L_2 + M(z_1)}\right] \tag{5}$$

Equation (5) of logarithmic function becomes the equation between their variables with $I_1 = I_0$

$$\frac{I_0 + I_2(z_2)}{I_0 + I_2(z_1)} = \frac{L_2 + M(z_1)}{L_2 + M(z_2)}$$

$$I_2(z_2) = \frac{1}{L_2 + M(z_2)}\left[I_0\{M(z_1) - M(z_2)\} + I_2(z_1)\{L_2 + M(z_1)\}\right] \tag{6}$$

The interaction force between the HTSC coil and magnet is given by the virtual work for the virtual displacement dz :

$$F = I_1 I_2 \frac{\partial M}{\partial z} \tag{7}$$

3. NUMERICAL ANALYSES

Flow of Analyses

The mutual inductance M of (4) is computed by the complete elliptic integrals of first and second kind

$$M = \mu_0 \sqrt{r_c r_p}\left\{\left(\frac{2}{k} - k\right) EllipticK(k^2) - \frac{2}{k} EllipticE(k^2)\right\} \tag{8}$$

where, μ_0 : vacuum permeability, r_c, r_p radius of the magnet and the HTSC coil, respectively, modulus k : $k = \sqrt{(4 r_c r_p)/\{(r_c + r_p)^2 + z^2\}}$. We can also calculate the derivative of M in (7) by the elliptic integrals of first and third kind

$$\frac{\partial M}{\partial z} = \frac{\mu_0(-z)}{4\sqrt{r_c r_p}} k^3 \left\{\left(\frac{2}{k^2} - 1\right) Elliptic\Pi(k^2, k^2) - \frac{2}{k^2} EllipticK(k^2)\right\} \tag{9}$$

We have computed the equations in the sequence from (8) (9) to (6) (7) by using Mathematica.

Computational Results

The parameters of the configuration shown by Fig. 1 are given as follows for the numerical analyses in reference to the experimental results by Prof. Tsuchimoto [1].

The permanent magnet: Nd-B-Fe, diameter 25 mm, thickness 22.5 mm, 1.12T magnetization

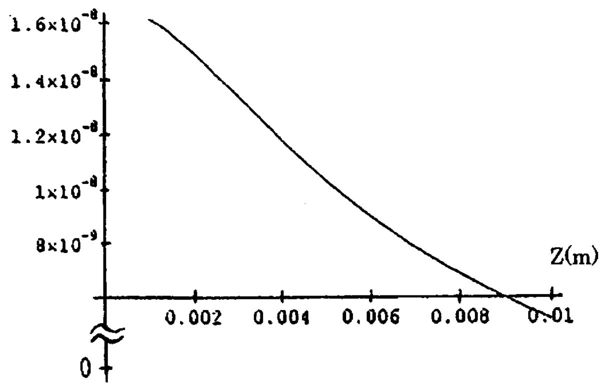
The bulk superconductor: Y-Ba-Cu-O Melt-Powder-Melt-Growth processed, diameter 18 mm, thickness 10 mm .

The mutual inductance together with the derivative between one turn coils which stand for the magnet with the radius $r_c = 12.5mm$, and the HTSC coil with the radius $r_p = 9mm$, have been calculated as the functions versus the distance between the coils by (8) and (9) as shown by Figures 3 and 4 .

The interactive force between the two coils calculated by (7) is shown by Figure 5, together with the HTSC coil current Figure 6. The interactive force becomes levitation when the magnet is approached, while departed becomes attraction. This phenomenon is the result of the electromechnical energy conversion related to the velocity emf, where the radial component of the magnetic fields interlinked with the HTSC coil are oriented outward both for the approach and departure due to the large magnetomotive force of the magnet.

Because of the linear differential equation (4), the coil current I_2 has unique solution for the distance z decrease and increase. While the nonlinearity due to the critical current density of superconductivity is taken into account, the hysteretic feature was found [1].

The computations based on the elliptic integrals have been much faster than the results obtained by using Legendre polynomials [3].



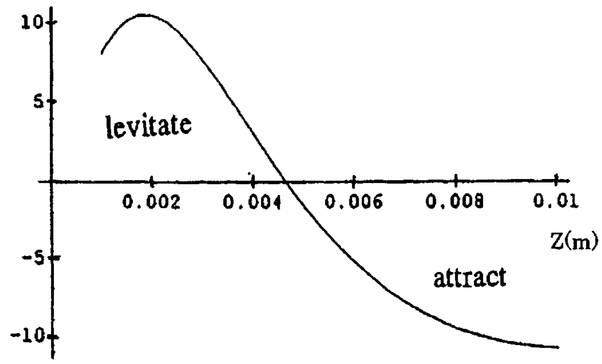
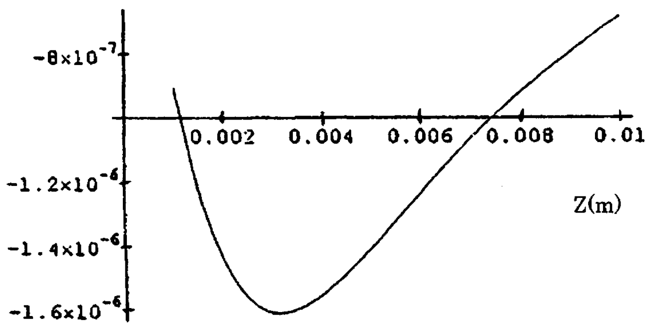


Figure 3. Mutual inductance between coils [H].Figure 5. Interactive force between coils [N].



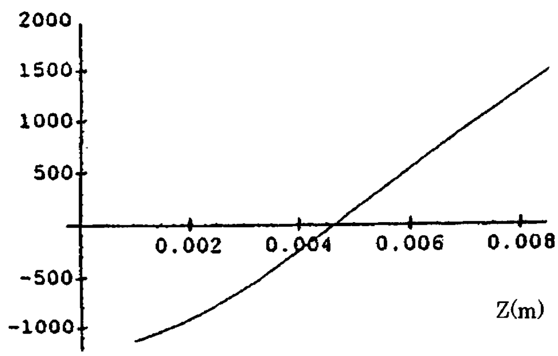


Figure 4. Derivative of mutual inductance [H/m].

Figure 6. HTSC coil current[A].

REFERENCES

1.Tsuchimoto,etal An axisymmetric boundary element analysis of levitation force on high Tc superconductor, Engineering Analysis with Boundary Elements, (1993), 11,171-175

2.H.H.Woodson,J.R.Melcher,ElectromechanicalDynamicsI,Appendix.B,eqs.(B.4.3,13), Wiley(1968)

3.Murakami Y. Mori K., Characterization of magnetic levitation of high Tc superconductor by energy conversion between electromagnetic and mechanical energy, Proc. International Conf. Electric Machines and Systems、 Shenyang, China (2001), 408-411

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