Loci of a Rotor Consisting of Disc and Shaft with the Moment of Inertia During Deceleration Through a Critical Speed

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A theoretical study of the vibration during deceleration through a critical speed with consideration of the moment of inertia has been made in this study. Loci of a rotor consisting of a disc and shaft with the moment of inertia during deceleration through the first critical speed is calculated numerically taking into account the moment of inertia, so that the following conclusion is obtained : The rotor can pass the critical speed during deceleration in no dumping system in case of finite moment of inertia.

1. Introduction

In conventional studies of the vibration during deceleration through a critical speed of a rotor consisting of a disc and shaft supported by self aligning ball bearings, the supports are assumed to have infinite moments of inertia [1-4]. However, in practice there are many cases of supports with a finite moment of inertia.

A theoretical study of the vibration during deceleration through a critical speed with consideration of the moment of inertia has been made in this study. This paper studies the case of deceleration through the first critical speed taking into consideration the moment of inertia.

2. Equations of motion

A rotor with an eccentricity, and consisting of a disc and a shaft supported having the finite moment of inertia rotates at a high speed (Fig.1).

Assumptions :

(1) The disc is fixed at the center of the shaft, and is free of gyroscopic moment.

(2) The mass of the shaft is negligible compared with the mass of the disc.

(3) Atmospheric influence is negligible.

(4) The effect of material hysteresis can be neglected and bearing friction is negligible.

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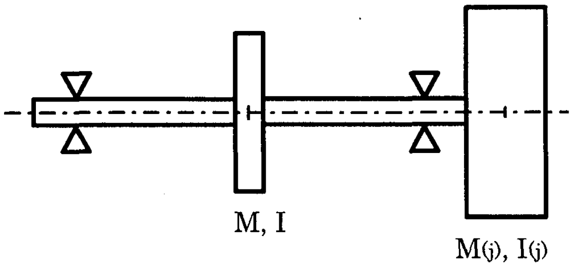
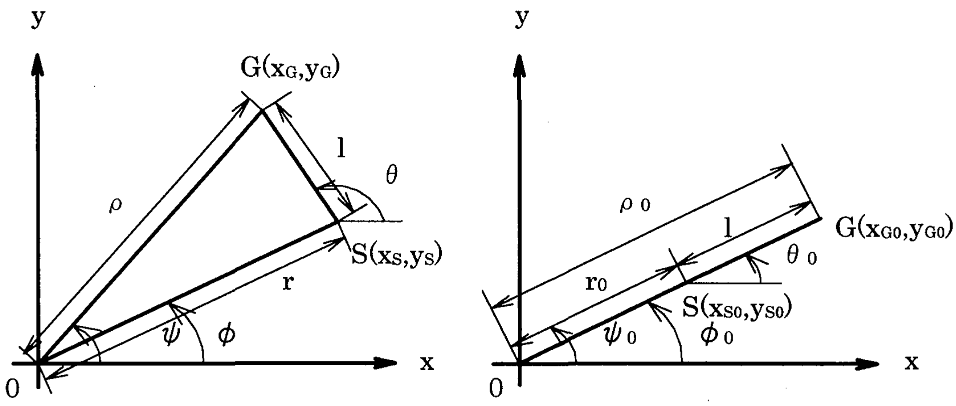


F1g.1　System layout



(a) General condition　　(b) Initial condition

Fig.2　System coordinates

The rotor is decelerated from the initial conditions shown in Fig.2(b) to the condition in Fig.2(a) which are based on a fixed co-ordinate system centered on 0.

The effect of moment of inertia of the support is included and dimensionless equations for the center of gravity of the rotor are obtained as follows :

$$\left.\begin{aligned}
&\frac{d^2X_G}{d\tau^2} = \cos\theta - X_G \\
&\frac{d^2Y_G}{d\tau^2} = \sin\theta - Y_G \\
&\frac{d^2\theta}{d\tau^2} = I^*(Y_G\cos\theta - X_G\sin\theta) \\
&I^* = I/\{\ I + (I^2/M)(I_{(j)}/I)(1/l^2)\ \}
\end{aligned}\right\} \quad (1)$$

Where t, X_G, Y_G are obtained as follows :

$$\left.\begin{aligned} \tau &= \omega_n t \\ X_G &= x_G / l \\ Y_G &= y_G / l \end{aligned}\right\} \quad (2)$$

Where ω_n is frequency of natural vibration with no damping.

Dimensionless equations for the initial conditions of the rotor are obtained as follows :

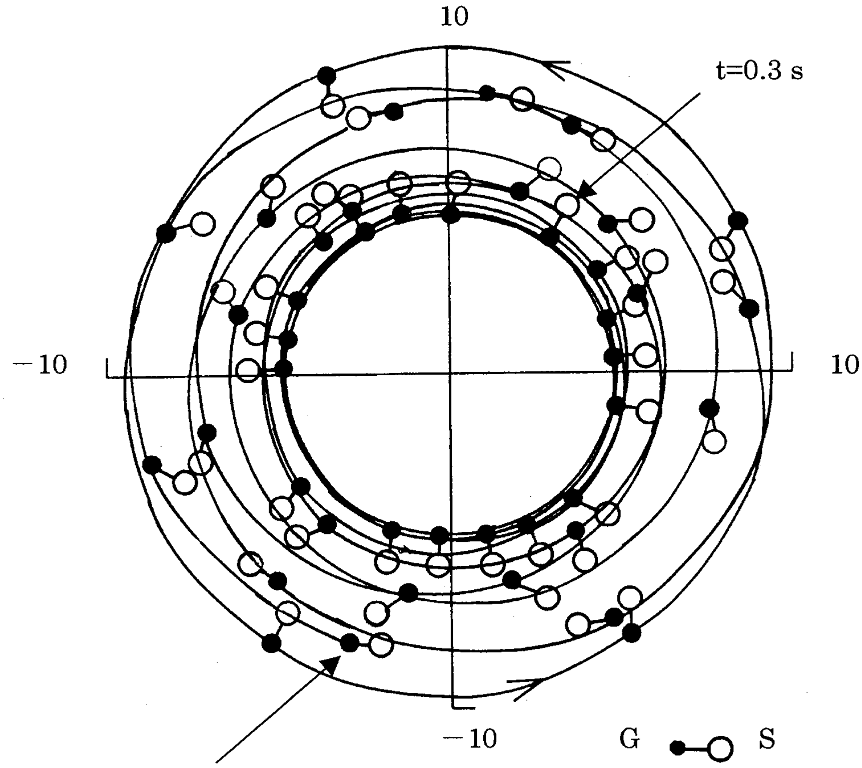
$$\left.\begin{aligned} R_0 &= \frac{d\Theta_0^2}{d\tau} / \left(\frac{d\Theta_0^2}{d\tau} - 1\right) \\ P_0 &= 1 / \left(\frac{d\Theta_0^2}{d\tau} - 1\right) \\ X_{G0} &= P_0 \cos\psi_0 \\ Y_{G0} &= P_0 \sin\psi_0 \\ X_{S0} &= X_{G0} - \cos\theta_0 \\ Y_{S0} &= Y_{G0} - \sin\theta_0 \\ \frac{dX_{G0}}{d\tau} &= -Y_{G0} \frac{d\psi_0}{d\tau} \\ \frac{dY_{G0}}{d\tau} &= X_{G0} \frac{d\psi_0}{d\tau} \\ \Theta_0 &= \pi + \phi_0 \\ \psi_0 &= \phi_0 \\ \frac{d\theta_0}{d\tau} &= \frac{d\psi_0}{d\tau} = \frac{d\phi_0}{d\tau} \\ \frac{d\theta_0}{d\tau} &= \omega_n \frac{d\Theta_0}{d\tau} \\ \omega_n &= 50cps \\ \frac{d\Theta_0}{d\tau} &= 1.1 \\ \phi_0 &= 0 \end{aligned}\right\} \quad (3)$$

Equations (1) are integrated numerically using the Runge Kutta method with the initial conditions from equations (3) . The results are them substituted in equations (4) to determine the remaining unknowns.

$$\left.\begin{aligned}
&X_S = X_G - \cos\theta \\
&Y_S = Y_G - \sin\theta \\
&P = \sqrt{(X_G^2 + Y_G^2} \\
&R = \sqrt{(X_S^2 + Y_S^2} \\
&\psi = \tan^{-1}\left(\frac{Y_G}{X_G}\right) \\
&\phi = \tan^{-1}\left(\frac{Y_S}{X_S}\right) \\
&\Psi = \frac{\psi}{2\pi} \\
&\Phi = \frac{\phi}{2\pi} \\
&\frac{dX_S}{d\tau} = \frac{dX_G}{d\tau} + \sin\theta\frac{d\theta}{d\tau} \\
&\frac{dY_S}{d\tau} = \frac{dY_G}{d\tau} - \cos\theta\frac{d\theta}{d\tau} \\
&\frac{dP}{d\tau} = \frac{dX_G}{d\tau}\cos\psi + \frac{dY_G}{d\tau}\sin\psi \\
&\frac{dR}{d\tau} = \frac{dX_S}{d\tau}\cos\phi + \frac{dY_S}{d\tau}\sin\phi \\
&\frac{d\psi}{d\tau} = \left(\frac{dY_G}{d\tau}\cos\psi - \frac{dX_G}{d\tau}\sin\psi\right)/P \\
&\frac{d\phi}{d\tau} = \left(\frac{dY_S}{d\tau}\cos\phi - \frac{dX_S}{d\tau}\sin\phi\right)/R \\
&\frac{d\Theta}{d\tau} = \frac{1}{\omega_n}\frac{d\theta}{d\tau} \\
&\frac{d\Psi}{d\tau} = \frac{1}{\omega_n}\frac{d\psi}{d\tau} \\
&\frac{d\Phi}{d\tau} = \frac{1}{\omega_n}\frac{d\phi}{d\tau}
\end{aligned}\right\} \quad (4)$$

3. Loci of the center of gravity of the rotor and the center of disc

Typical loci of the center of gravity of the rotor and the center of disc during deceleration through a critical speed for the value of $I^* = 10^{-2}$ are shown in F1g.3. The marks ● and ○ in Fig.3 are the center of gravity of the rotor and the center of disc respectively. It can be seen from Fig.3 that the rotor can pass the critical speed during deceleration in no dumping system in case of finite moment of inertia.



$(I^* = 10^{-2}, \quad t = 0.3s - 0.5s)$

Fig.3 Loci of the center of gravity of the rotor and the center of disc through a critical speed

4. Conclusions

Loci of a rotating disc during deceleration through the first critical speed are calculated numerically taking into account the moment of inertia, so that the following conclusions are obtained : The rotor can pass the critical speed during deceleration in no dumping system in case of finite moment of inertia.

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References

1. I.M. Hall & E.W.E. Rogers, Part I, The Flow Pattern on a Tapered Sweptback Wing at Mach Numbers between 0.6 and 1.6, Part II, Experiments with a Tapered Sweptback Wing of Warren 12 Plan form at Mach Numbers between 0.6 and 1.6, *Re p.& Mem., Aeron. Re s. Council,* No.3271 (1960).
2. R.F.Meyer, A Note on a Technique of Surface Flow Visualization, *Nat. Re s. Coun. Canada,* Aero-Rep., LR-457 (1966).
3. G.D. Raithby & E.R.G. Eckert, The Effect of Support Position and Turbulence Intensity on the Flow Near the Surface of a Sphere, *Wärme-und Staffübertragung,* 1 (1968), 87.
4. K. Hironaka, Flow in the Deria Pump turbine, *Proc.2nd Inc. Japan. Soc. Mech. Engr. Symp.,* 1 (1972), 13g-145.

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