

# Fourier and Wavelet Analyses of Normal and Epileptic Electroencephalogram

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**Abstract:** Human normal and epileptic electroencephalogram (EEG) signals have been analyzed using Fourier Transform (FT) and Continuous Wavelet Transform (CWT) as well as Discrete Wavelet Transform (DWT). The power spectrum densities of both normal and epileptic EEG showed  $1/f^a$  fluctuations as a function of frequency  $f$ . The alpha rhythms that appeared in the normal EEG were not observed in the epileptic EEG. FT could not reveal the localized spikes and complexes that are typical among epileptic seizures. On the contrary, both CWT and DWT analyses could clearly detect localized low-frequency epileptic components in the EEG records. Our final goal of the study is the automatic detection of the epileptic disorders in the EEG in order to support the diagnosis and care of the epileptic syndromes and related seizure disorders.

**Keywords** - Electroencephalogram, EEG, epileptic seizure, Fourier transform, wavelet transform, signal analysis,  $1/f^a$  fluctuation

## I. INTRODUCTION

The electroencephalogram (EEG) is a record of a time series of evoked potentials caused by systematic neural activities in a brain. The measurements of the human EEG signals are performed through electrodes placed on the scalp, and they are usually recorded on paper against time. The voltage of the EEG signal corresponds to its amplitude. The typical amplitudes of the scalp EEG lie between 10 and 100  $\mu$  V, and in adults more commonly 10 and 50  $\mu$  V.

The frequency range of the EEG extends from ultra-slow to ultra-fast frequency components that play no significant role in the clinical EEG. Clinically meaning frequencies lie between 0.1 Hz and 100Hz. In more restricted sense, the frequency range is classified into several frequency components, or  $\delta$  rhythm (0.5~<4Hz),  $\theta$  rhythm (4~<8Hz),  $\alpha$  rhythm (8~<13Hz),  $\beta$  rhythm (13~<30Hz), and  $\gamma$  rhythm (30~<60Hz). In the normal adult slow components,  $\delta$  and  $\gamma$  rhythms, are sparsely represented, and fast components,  $\alpha$  and  $\beta$  rhythms predominate[1].

It has been recognized that certain forms of epileptic seizure disorders have special clinical and EEG characteristics. Epileptic syndromes are age-dependent and chiefly occur in certain age range, especially infancy and childhood, but frequently persisting through adolescence into adult life [1]. Types of the epileptic seizures and epileptic syndromes are classified by the Commission on Classification and Terminology of the International League Against Epilepsy [2].

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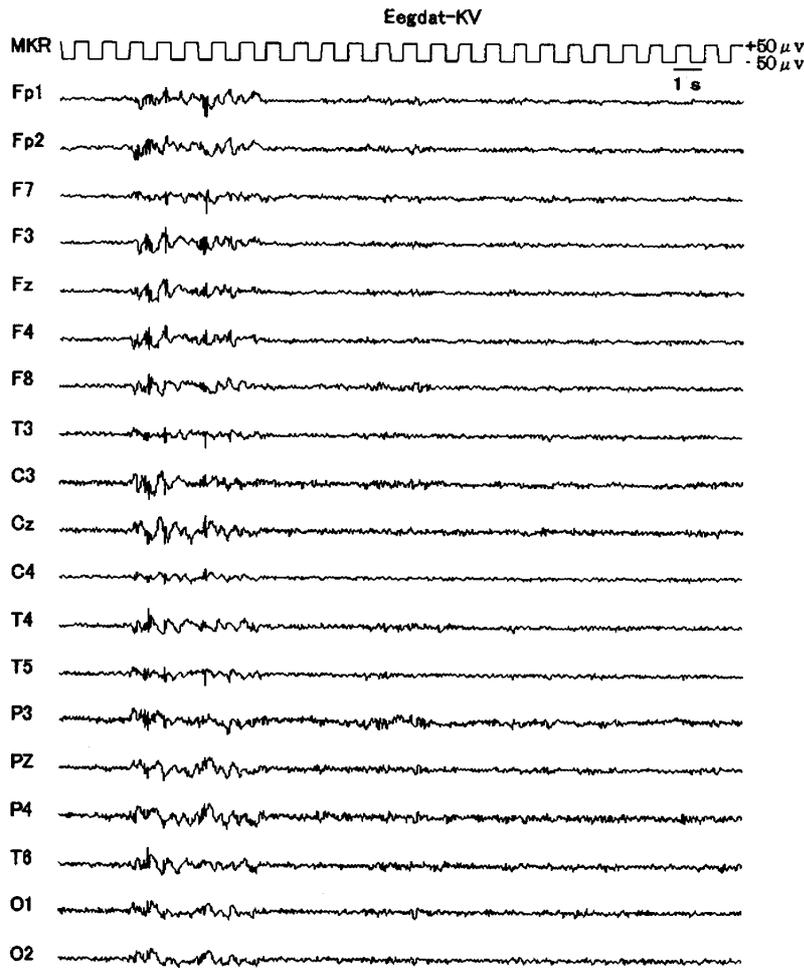


Fig. 1. Epileptic EEG measured for 25s. Amplitude of the marker is  $\pm 50 \mu V$ , after K.Krajca[4].

In this paper, we have analyzed the human normal and epileptic EEG signals from the stand point of the wave form and periodicity using the Fourier transform (FT) and the wavelet transform in order to test their abilities to detect localized characteristic frequency components in EEG.

## II. EEG data

The normal adult EEG signal data have been provided by T. Musha of the Brain Functions Laboratory [3], and the typical

epileptic EEG signal data by V. Krajca from Faculty Hospital Bulovka, Czech [4]. Both of the EEG data have been measured with the electrodes placed on the scalp according to the international 10-20 system with both earlobes chosen as common referential electrodes. Normal EEG data have been measured at a sampling frequency of 100Hz for about 82 sec, giving 8192 samples for each channel.

Fig. 1 shows the epileptic EEG signals that have been recorded at a sampling frequency of 128Hz for 25 sec, giving 3200 samples for each channel. The marker signal (MKR) has the amplitude of  $\pm 50 \mu s$ . Analyses of the EEG signals have been done for segments and all of the recorded signals.

## III. RESULTS

### A. Fourier Transform

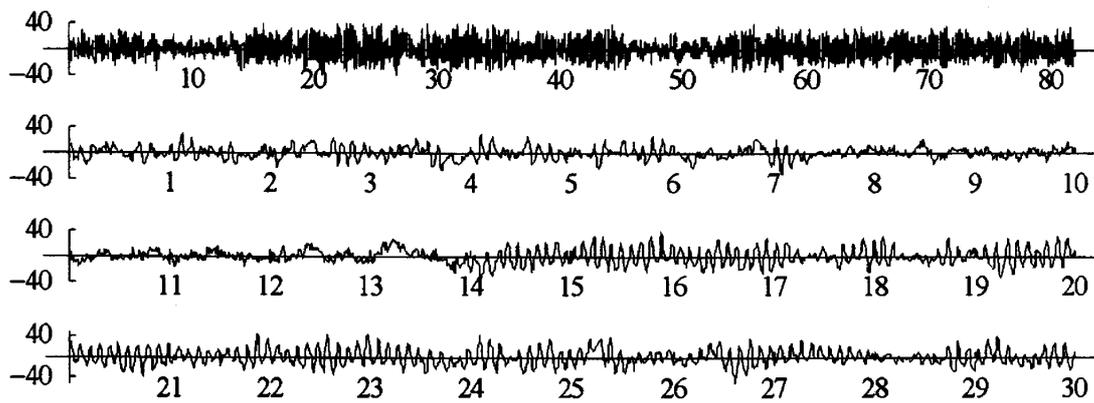


Fig.2. Normal EEG recorded from C3 electrode with its segmental details. Amplitude is in  $\mu$  V, after Musha[3].

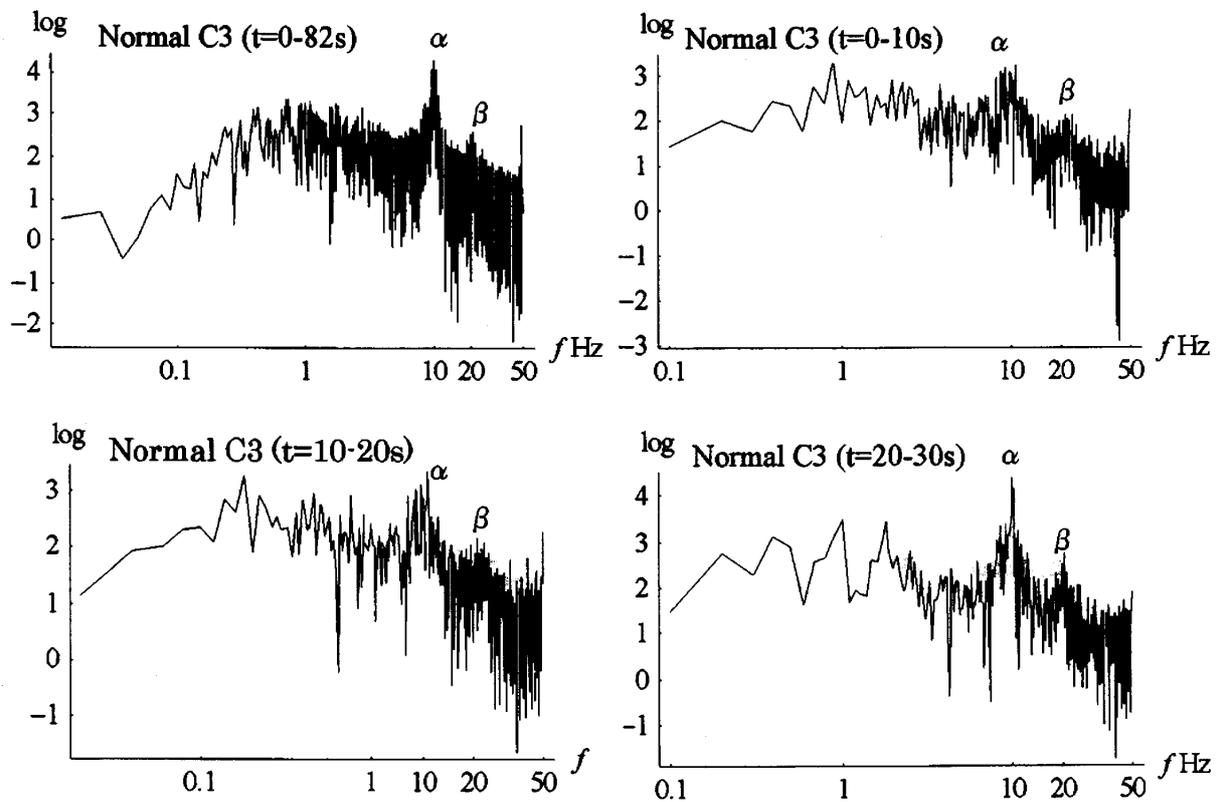


Fig. 3. Power spectrum densities for various normal EEG segments recorded from C3 electrode. Alpha and beta peaks are clearly identified.

As one of the typical examples of the normal EEG, Fig. 2 shows the EEG signals recorded from the C3 electrode and its three short (10-seconds) segments. The horizontal axis is a time base in seconds and the longitudinal axis shows the signal amplitudes in  $\mu$  V. Fig. 3 depicts the power spectrum densities (PSD) for the corresponding signals given in Fig. 1. PSD is expressed as the square of the absolute values of the Fourier transform. In each PSD so called  $1/f^\alpha$  fluctuation can be seen as a function of frequency  $f$ . The peaks that indicate alpha and beta rhythms can be

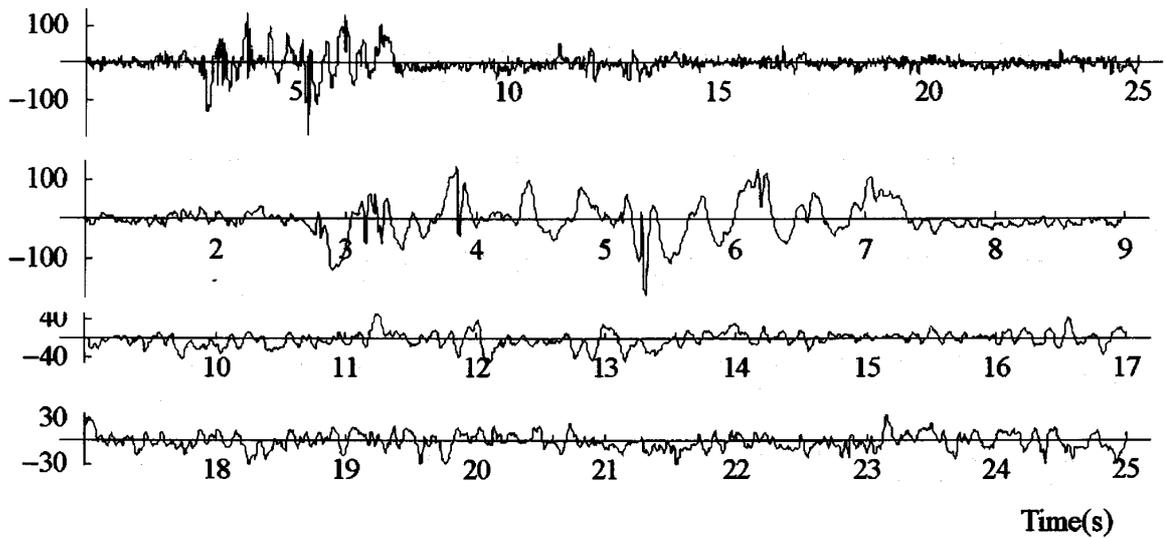


Fig.4. Epileptic EEG recorded from Fp1 electrode with its segmental details. Amplitude is in  $\mu$  V.

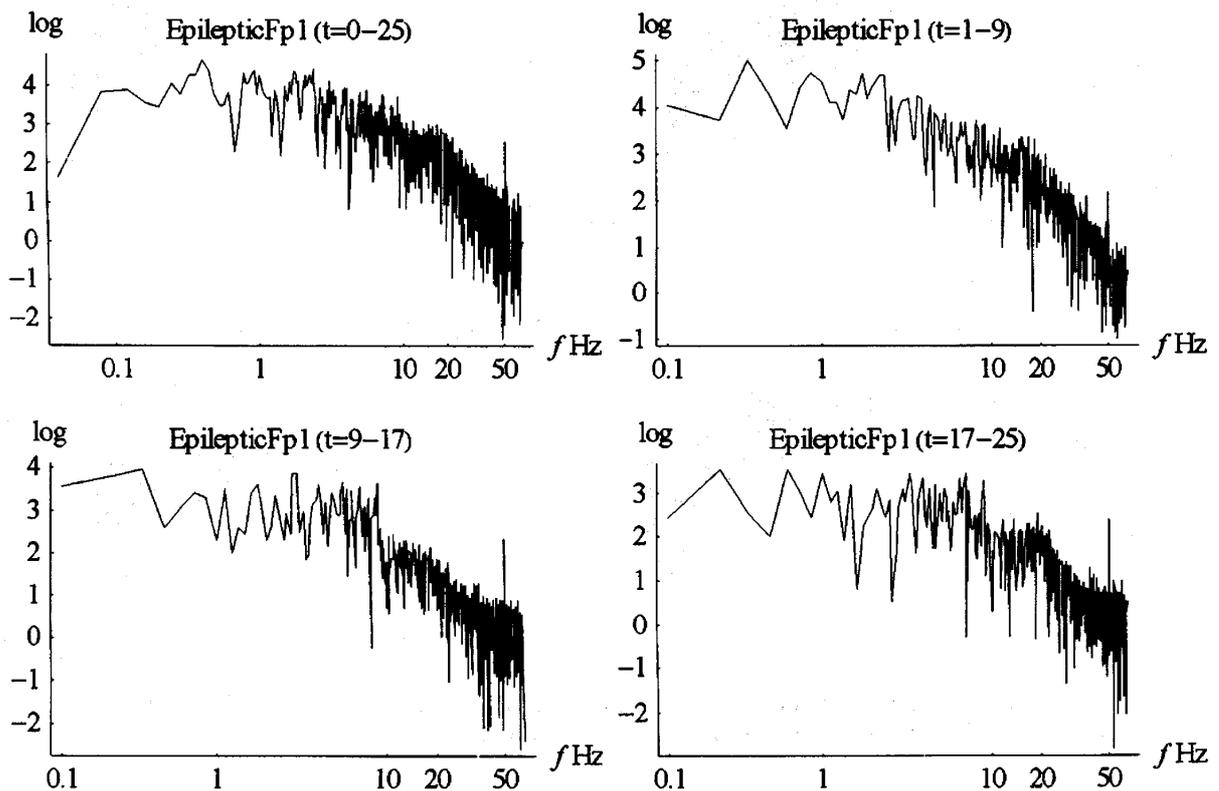


Fig. 5. Power spectrum densities for various epileptic EEG segments recorded from Fp1 electrode.

seen clearly in each PSD spectrum, although less clearly in PSD for the time interval 0-10s. Other frequency components ( $\delta$ ,  $\theta$  and  $\gamma$ ) are not appeared in the PSD spectra. The same tendency, the  $1/f^\alpha$  fluctuation and appearance of alpha and beta rhythms, has been observed for the EEG signals from other channels, although the

figures are not shown.

The same analyses have been performed for the epileptic EEG records. Fig.4 shows a typical example of the EEG signal that was recorded from Fp1 electrode for 25s along with its three short (8-seconds) segments. As in the normal EEG, the horizontal axis is a time base in seconds and the longitudinal axis shows the signal amplitudes in  $\mu V$ . The occurrence of the epileptic seizure can be seen in 2s after the beginning of the measurement. Fig. 5 depicts the power spectrum densities (PSD) for the corresponding signals given in Fig. 4. In all PSD spectra, neither alpha nor beta rhythms the normal EEG, and absence of the alpha rhythms in the epileptic EEG is understandable. Moreover, the slow frequency components that are characteristic in the epileptic seizure are detected, neither, in the PSD for the time interval of 1-9s. The same phenomena have been observed in the PDSs for the EEG from other electrode, e.g. P3 and O1.

### B. Wavelet Transform

Continuous wavelet transform (CWT) of an analog signal  $f$  is expressed as [5]:

$$C_{b,a} := (W_{\psi} f)(b,a) := |a|^{-\frac{1}{2}} \int_{-\infty}^{\infty} f(t) \overline{\psi\left(\frac{t-b}{a}\right)} dt. \quad (1)$$

Function  $\psi((t-b)/a)$  is obtained by the translation  $b$  and the dilation  $a$  (the scale factor) of the "basic wavelet" (or "mother wavelet")  $\psi(t)$ . The result of the CWT is many wavelet coefficients  $C_{b,a}$ . In order to construct efficient algorithms for calculating the wavelet transform  $W_{\psi} f(b,a)$ , dyadic values  $b = k/2^j$  and  $a = 2^{-j}$  are usually chosen. This way of the wavelet transform is called the Dyadic Wavelet Transform or the Discrete Wavelet Transform (DWT). In DWT,  $C_{b,a}$  in equation (1) is given as

$$C_{b,a} := (W_{\psi} f)\left(\frac{k}{2^j}, \frac{1}{2^j}\right). \quad (2)$$

### C. Wavelet Analysis

Fig. 6(a) depicts a part (1-9s interval) of the epileptic EEG signals measured from the Fp1 electrode. Fig. 6(b) shows the CWT coefficients analyzed with Daubechies wavelet of

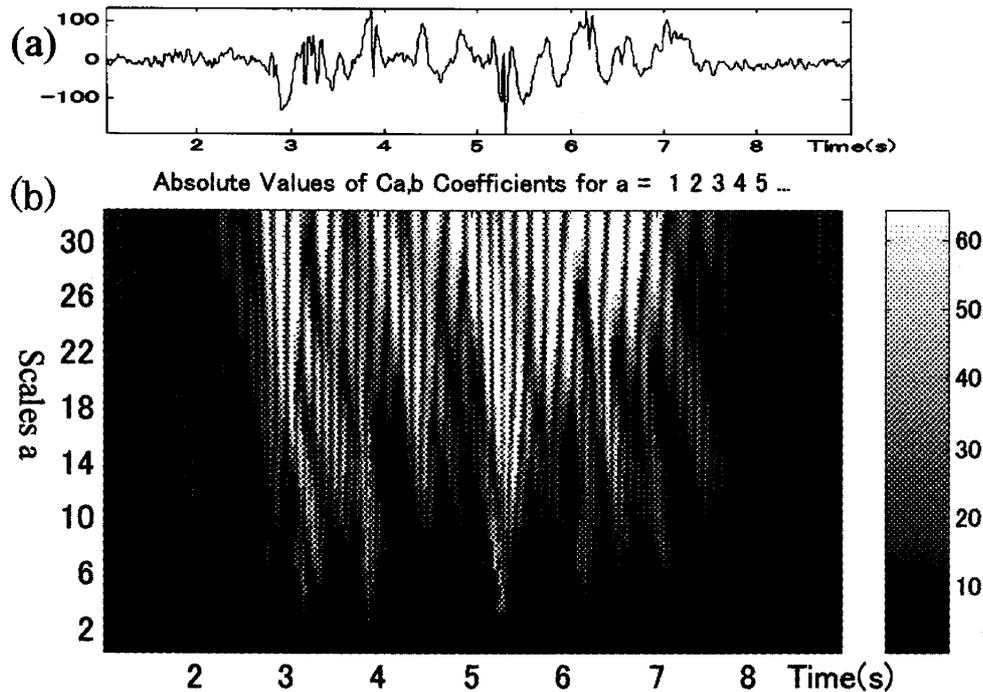


Fig. 6. (a) Epileptic EEG from Fp1; (b) absolute values of CWT wavelet coefficients as a function of scale factor  $a$  with 8<sup>th</sup> order Daubechies wavelet.

order 8. Scale level (the longitudinal axis) was incrementally changed from 1 (higher frequency) to 32 (lower frequency) in every single step. The larger the values of  $a$ , the slower frequency components. The horizontal axis gives the sampling number, or time in seconds. The gray scale index corresponds to the values of the wavelet coefficients. The slow frequency components of the EEG signals are well detected. This is more clearly seen in Fig. 7 that shows the results by DWT with Daubechies wavelet of order 8 for the epileptic EEG signals from the Fp1 electrode. Wavelet decomposition was continued down to the level of 5. Detailed parts (D1-D5) show the higher frequency components at each decomposition level, while A5 gives the lower frequency component at the 5<sup>th</sup> level. Original signal  $S$  is reconstructed by adding all the components, i.e.  $S = A5 + D5 + D4 + D3 + D2 + D1$ . The local low frequency waves concerned with the epileptic seizure in the EEG are clearly detected by DWT.

#### IV. DISCUSSION

The normal and epileptic EEG records have been analyzed with Fourier Transform and Continuous Wavelet Transform. Multi level decomposition of the EEG signal has also been performed using Discrete Wavelet Transform. In the PSD for the normal EEG signals, the alpha and beta rhythms have

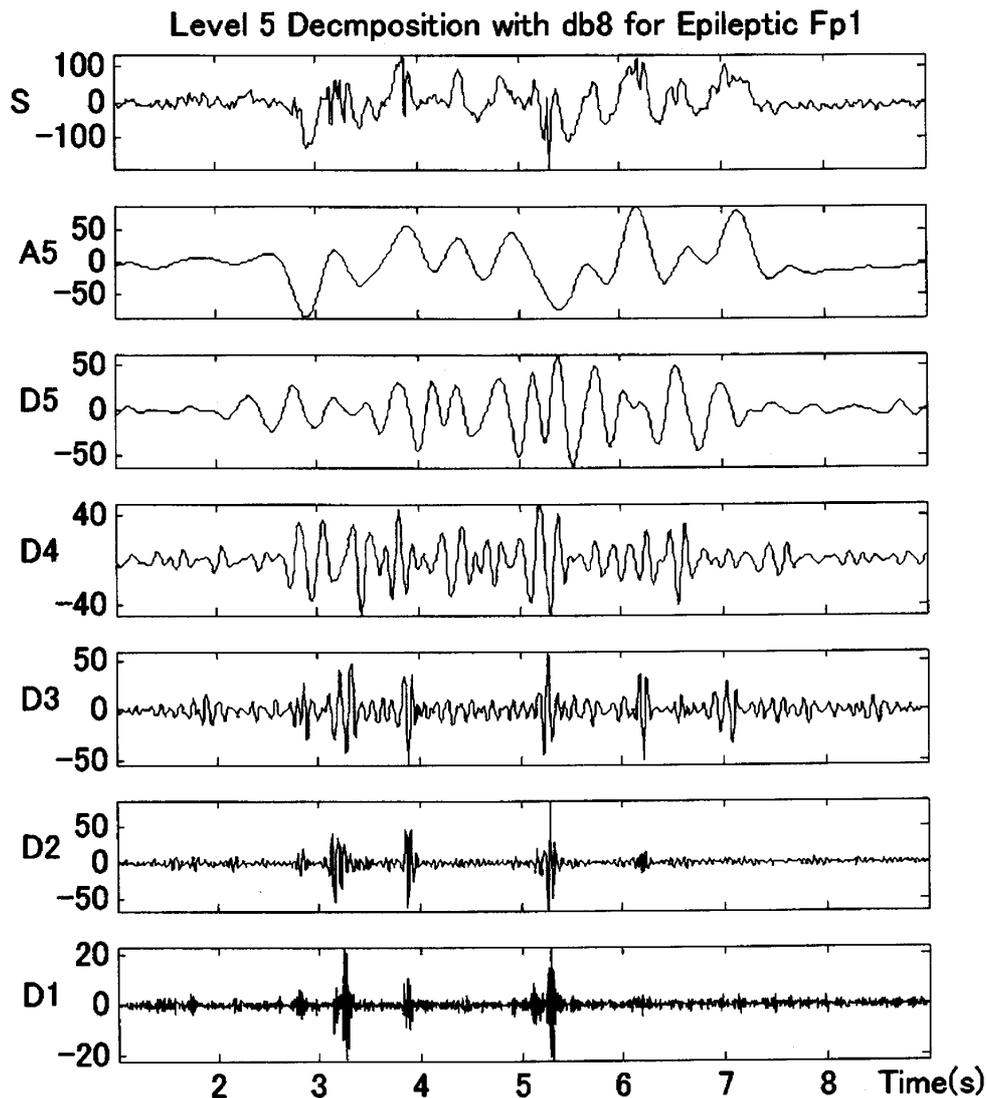


Fig. 7. Fifth level decomposition of epileptic EEG signal (S) from Fp1 electrode with 8<sup>th</sup> order Daubechies wavelet. Detailed parts (D1-D5) show higher frequency components and approximate part A1 lower frequency component. The longitudinal axis shows amplitudes in  $\mu$  V.

been clearly appeared. On the contrary, neither alpha nor beta rhythms have been observed in the PSD for the epileptic EEG, which is consistent with the fact that alpha rhythms are disappeared in the epileptic EEG record. The  $1/f^2$  fluctuation has been observed in all PSDs. The local low frequency components in each normal and epileptic EEG have been clearly detected by Wavelet transform, but not by Fourier transform. Fourier transform is a well-established technique for analyzing various periodic signals. This technique, however, is not appropriate for analyzing non-periodic biosignals such as EEG. We conclude that wavelet transform is well suited for local time-frequency analysis of the EEG signals.

Automatic detection of epileptic seizures in EEG and its diagnostic application are our next work.

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