Vibration Characteristics of Rotor Consisting of Disc and Shaft with Moment of Inertia During Deceleration Through Critical Speed*

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A theoretical study of the vibration during deceleration through a critical speed with consideration of the moment of inertia has been made in this study. Vibration characteristics of a rotor consisting of a disc and shaft with the moment of inertia during deceleration through the first critical speed is calculated numerically taking into account the moment of inertia, so that the following conclusion is obtained : The rotor can pass the critical speed during deceleration in no damping system in case of finite moment of inertia.

Key Words : Critical speed , Rotor dynamics, Vibration

1. Introduction

In conventional studies of the vibration during deceleration through a critical speed of a rotor consisting of a disc and shaft supported by self aligning ball bearings, the supports are assumed to have infinite moments of inertia[(1)~(4)]. However, in practice there are many cases of supports with a finite moment of inertia.

A theoretical study of the vibration during deceleration through a critical speed with consideration of the moment of inertia has been made in this study. This paper studies the case of deceleration through the first critical speed taking into consideration the moment of inertia of the support.

2. Equations of motion

A rotor with an eccentricity, and consisting of a disc and a shaft supported having the finite moment of inertia rotates at a high speed (Fig.1).

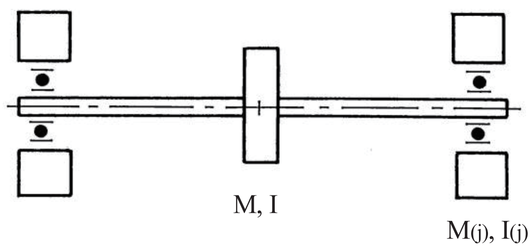
Assumptions :

(1) The disc is fixed at the center of the shaft, and is free of gyroscopic moment.

(2) The mass of the shaft is negligible compared with the mass of the disc.

(3) Atmospheric influence is negligible.

(4) The effect of material hysteresis can be neglected and bearing friction is negligible.



F1g.1 System layout

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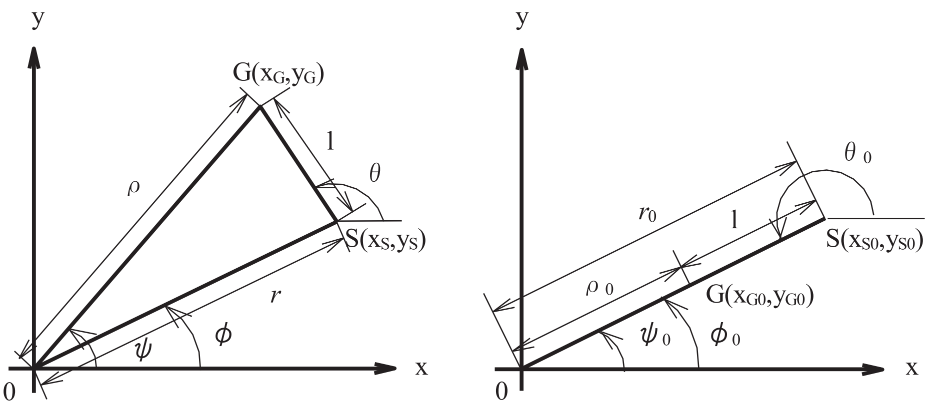

(a) General condition (b) Initial condition

Fig.2 System coordinates

The rotor is decelerated from the initial conditions shown in F1g.2(b) to the condition in F1g.2(a) which are based on a fixed co-ordinate system centered on 0.

The general condition of a rotating disc is shown in Fig.2(a). In Fig. 2 (a), O, S, and G indicate the origin of coordinates, the geometric center of the disc, and the center of gravity of the disc respectively. G is offset by an eccentricity l from S. In the case of the disc is not rotating and the shaft is not bending, S coincides with the center of the shaft and the origin O of the coordinates. In the case of the disc rotating, the position from O to S and G changes. At this time, the positions from O to S and G are indicated as polar coordinates (r, ϕ) and (ρ, ψ) respectively.

The effect of moment of inertia of the support is included and dimensionless equations for the center of gravity of the rotor are obtained as follows：

$$\left.\begin{aligned} &\frac{d^2X_G}{d\tau^2} = \cos\theta - X_G \\ &\frac{d^2Y_G}{d\tau^2} = \sin\theta - Y_G \\ &\frac{d^2\theta}{d\tau^2} = I^*(Y_G\cos\theta - X_G\sin\theta) \\ &I^* = I/\{\ I + (I^2/M)(I_{(j)}/I)(1/l^2)\ \} \end{aligned}\right\} \quad (1)$$

Where τ, X_G, Y_G are obtained as follows：

$$\left.\begin{aligned} &\tau = \omega_n t \\ &X_G = x_G / l \\ &Y_G = y_G / l \end{aligned}\right\} \quad (2)$$

Where ω_n is frequency of natural vibration with no damping.

The rotor is decelerated from the initial conditions shown in F1g.2(b). Dimensionless equations for the initial conditions of the rotor are obtained as follows :

$$
\left.
\begin{aligned}
&R_0 = \frac{d\Theta_0^2}{d\tau} / (\frac{d\Theta_0^2}{d\tau} - 1) \\
&P_0 = 1/(\frac{d\Theta_0^2}{d\tau} - 1) \\
&X_{G0} = P_0 \cos\psi_0 \\
&Y_{G0} = P_0 \sin\psi_0 \\
&X_{S0} = X_{G0} - \cos\theta_0 \\
&Y_{S0} = Y_{G0} - \sin\theta_0 \\
&\frac{dX_{G0}}{d\tau} = -Y_{G0} \frac{d\psi_0}{d\tau} \\
&\frac{dY_{G0}}{d\tau} = X_{G0} \frac{d\psi_0}{d\tau} \\
&\Theta_0 = \pi + \phi_0 \\
&\psi_0 = \phi_0 \\
&\frac{d\theta_0}{d\tau} = \frac{d\psi_0}{d\tau} = \frac{d\phi_0}{d\tau} \\
&\frac{d\theta_0}{d\tau} = \omega_n \frac{d\Theta_0}{d\tau} \\
&\omega_n = 50cps \\
&\frac{d\Theta_0}{d\tau} = 1.1 \\
&\phi_0 = 0
\end{aligned}
\right\} \quad (3)
$$

Equations (l), (2) are integrated numerically using the Runge Kutta method with the initial conditions from equations (3). The results are them substituted in equations (4) to determine the remaining unknowns.

$$
\left.\begin{aligned}
&X_S = X_G - \cos\theta \\
&Y_S = Y_G - \sin\theta \\
&P = \sqrt{(X_G^2 + Y_G^2} \\
&R = \sqrt{(X_S^2 + Y_S^2} \\
&\psi = \tan^{-1}(\frac{Y_G}{X_G}) \\
&\phi = \tan^{-1}(\frac{Y_S}{X_S}) \\
&\Psi = \frac{\psi}{2\pi} \\
&\Phi = \frac{\phi}{2\pi} \\
&\frac{dX_S}{d\tau} = \frac{dX_G}{d\tau} + \sin\theta\frac{d\theta}{d\tau} \\
&\frac{dY_S}{d\tau} = \frac{dY_G}{d\tau} - \cos\theta\frac{d\theta}{d\tau} \\
&\frac{dP}{d\tau} = \frac{dX_G}{d\tau}\cos\psi + \frac{dY_G}{d\tau}\sin\psi \\
&\frac{dR}{d\tau} = \frac{dX_S}{d\tau}\cos\phi + \frac{dY_S}{d\tau}\sin\phi \\
&\frac{d\psi}{d\tau} = (\frac{dY_G}{d\tau}\cos\psi - \frac{dX_G}{d\tau}\sin\psi)/P \\
&\frac{d\phi}{d\tau} = (\frac{dY_S}{d\tau}\cos\phi - \frac{dX_S}{d\tau}\sin\phi)/R \\
&\frac{d\Theta}{d\tau} = \frac{1}{\omega_n}\frac{d\theta}{d\tau} \\
&\frac{d\Psi}{d\tau} = \frac{1}{\omega_n}\frac{d\psi}{d\tau} \\
&\frac{d\Phi}{d\tau} = \frac{1}{\omega_n}\frac{d\phi}{d\tau}
\end{aligned}\right\} \quad (4)
$$

3. Vibration characteristics of a rotor during deceleration through a critical speed

Vibration characteristics of a rotating disc during deceleration through a critical speed in case of $I^*=10^{-2}$ are shown in Fig.3. Where $d\Theta/d\tau=1$ in Fig.3(f) corresponds to ω_n which is the frequency of natural vibration with no damping. It is shown from Fig.3 that the rotor can pass the critical speed during deceleration in no dumping system in case of finite moment of inertia.

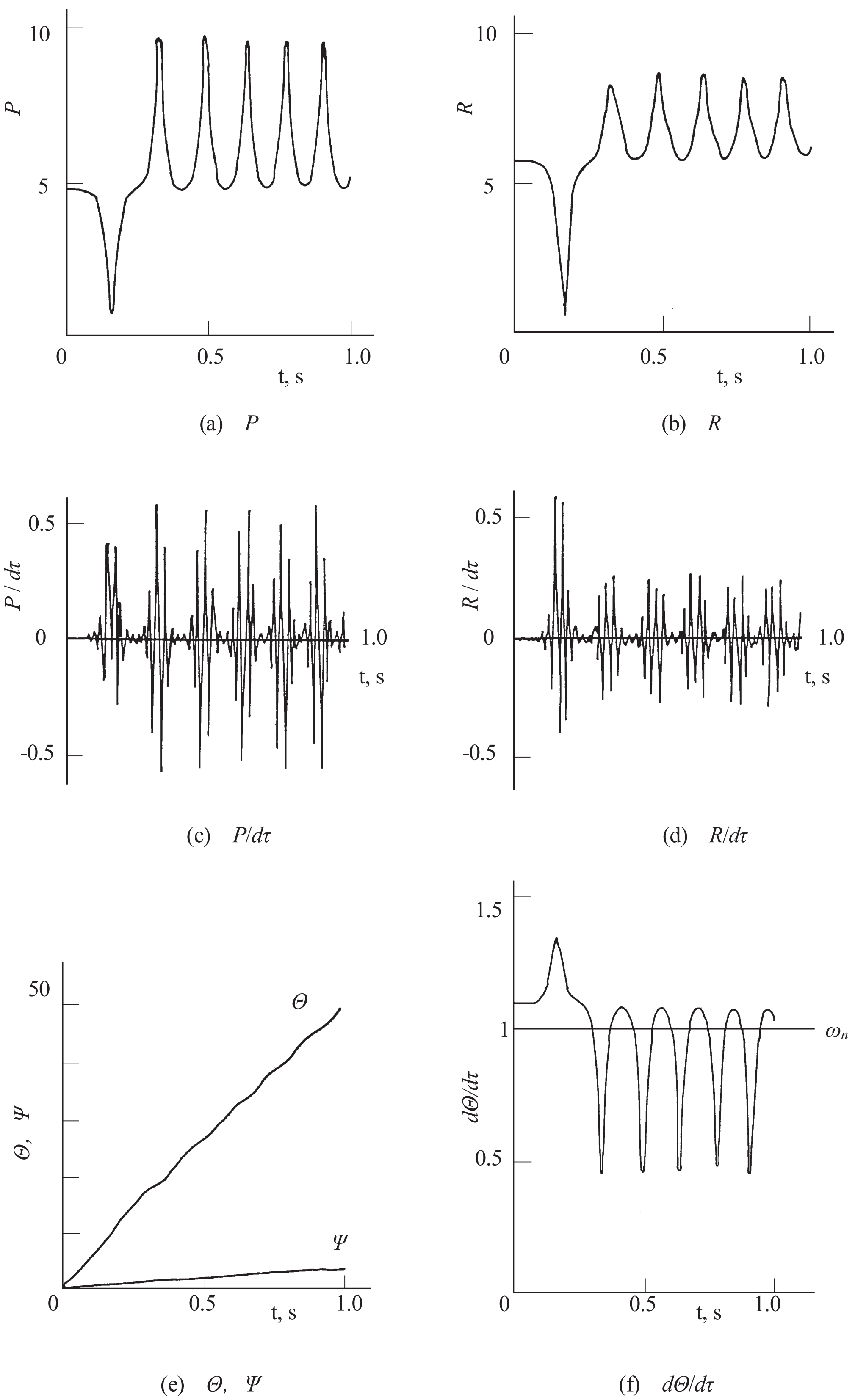

Fig.3 Vibration characteristics of a rotor in case of $I^* = 10^{-2}$

Typical dimensionless angular velocities of the center of gravity of the rotor for different values of I^* are shown in Fig.4(a)-(d). Fig4(b)-(d) show that the rotor can pass the critical speed during deceleration in no damping system when the moment of inertia of supports not so large to that of a disc. Fig.4(a) shows the dimensionless angular velocity of the center of gravity of the rotor in the infinite supports moment of inertias, and this corresponds to the conventional case. In this case the dimensionless angular velocity of the center of gravity of the rotor has a constant value, and the rotor cannot pass the critical speed.

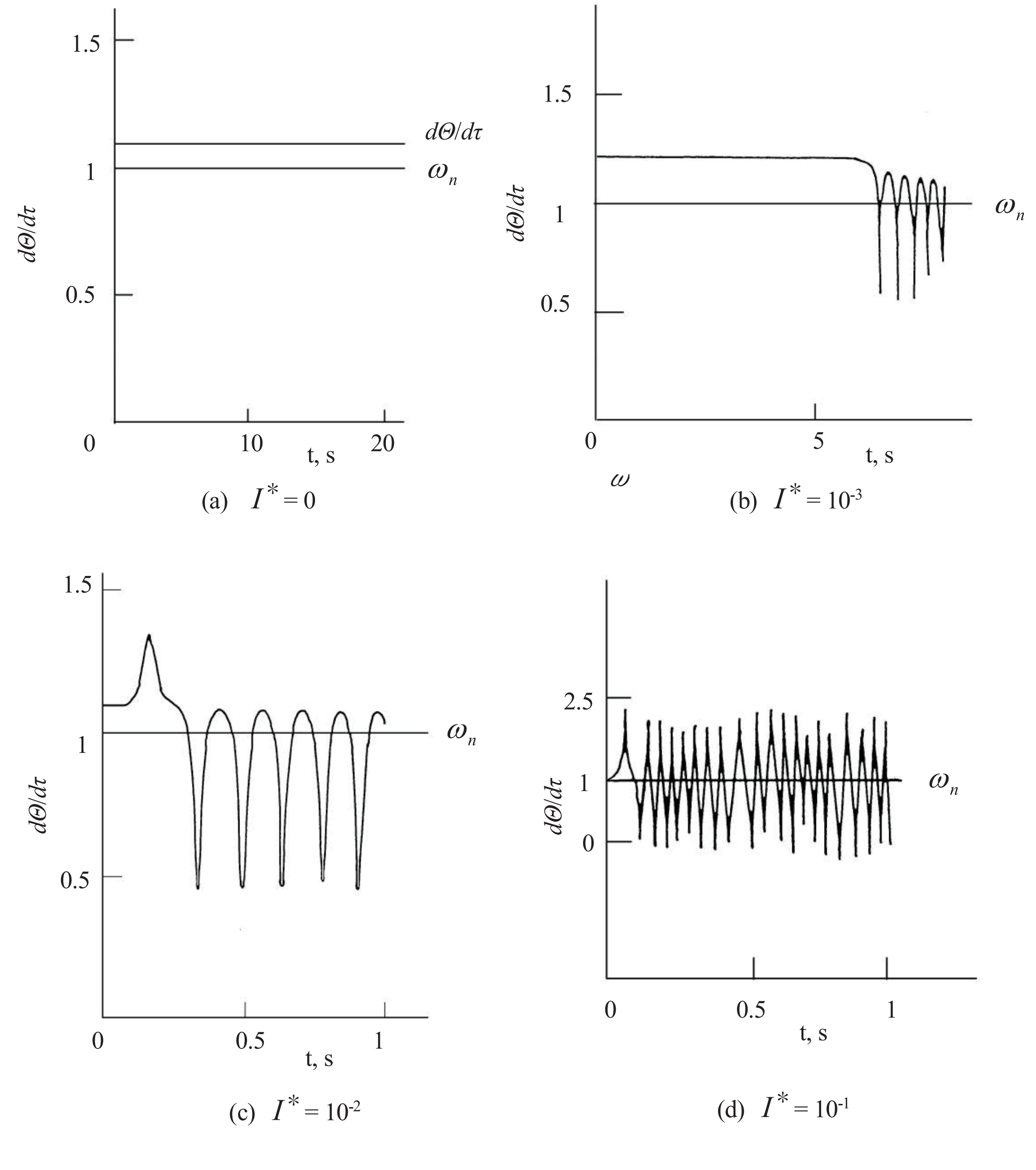

Fig.4 Typical dimensionless angular velocities of the center of gravity of the rotor

The minimum times required t^* for passing through the critical speed for different values I^* are shown by the curves in Fig.5. The curves shows that the greater I^* becomes, i.e. the smaller the moment of inertia of the supports, the smaller the value of t^* , hence the critical speed can be pass more easily.

Typical dimensionless maximum amplitudes of vibration of the center of the rotating disc are shown in Fig.6. It is shown in Fig.6 that the smaller the support moment of inertia i.e. the larger the value of I^* , the smaller the maximum amplitude of vibration of the center of rotating disc. It is assumed the support moment of inertia varies inversely with the restoring moment of the inertia force and the energy of rotation increase. Then as the energy of translation decreases in size, by the law of conservation of mechanical energy, the amplitude of vibration of rotating disc decreases in size.

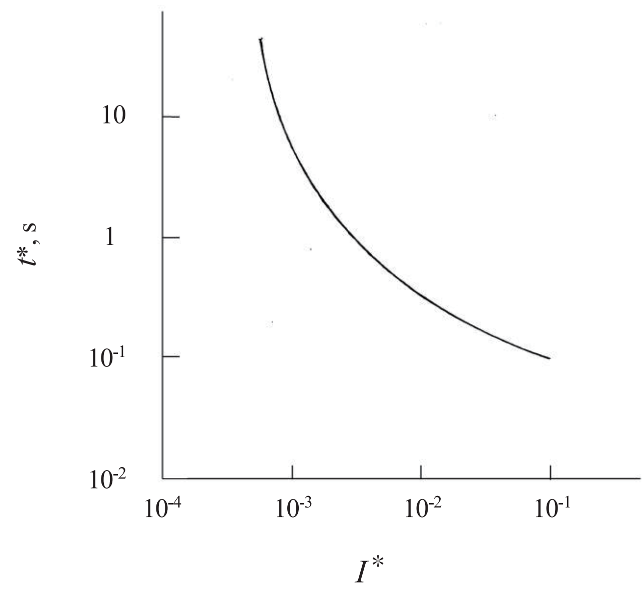

Fig.5 The minimum times required t^* for passing through the critical speed

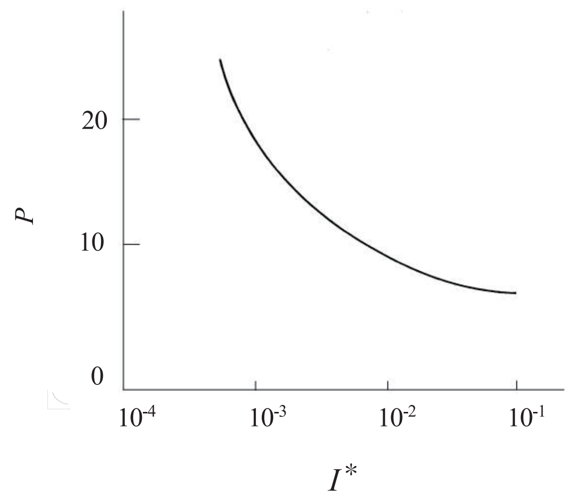

Fig.6 Typical dimensionless maximum amplitudes of vibration of the center of the rotating disc

4. Conclusions

Vibration characteristics of a rotating disc during deceleration through the first critical speed in no damping system are calculated numerically, so that the following conclusion is obtained.

(1) The rotor can pass the critical speed during deceleration in no damping system in case of finite moment of inertia.

(2) The lesser the support inertia, the smaller the maximum amplitude of vibration of the rotor when passing through the critical speed.

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