正弦カム曲線系における従動節の振動について

奥　田　　　薫

On the Vibration of the Followers in the Sine Cam Curve Series

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The objective points of the author are to give the practical vibration of the follower in the sive cam curue of the cam linkage system.

Much has been written from a theoritical standpoint about the various shapes and types of cams. However, in the authors' opinion, too little is the vibration study concerning of accurately the various cam series.

The author has found the general vibrating equatins of the sine profile cam having the damping factors acting between the frames and the followers and then followers and the vibrating weight and researched the effects of the changes of those values in the displacements, forces and pulses.

I. 緒　　言

本論文はカム振動系においてカム曲線として正弦曲線を使用した場合の従動節における振動状態を考察したものである。

このためにカム軸内の回転摩擦による減衰係数，フレームと従動節間の減衰係数を考慮し一般形における振動式を求めた。これを解き以上の減衰係数のある場合とない場合における変位，速度，加速度すなわち力関係およびパルスガ如何に変化するかを検討した。

II. 変 位 曲 線

カム装置の従動節端の相当重量 Wkg，その相当質量 m(kgs²/mm)，従動節先端の変位 y(mm)，カム変位 y_c (mm)，リング系の相当バネ定数 k (kg/mm)，拘束用バネ常数 k_s (kg/mm)，フレームと従動節間の減衰係数 c_f (kg/mm)，従動節内の減衰係数 c_l (kgs/mm)，初期のバネ力 s_1(kg) とする。

カムの曲線を sine 曲線とすると，この場の運動方程式は次のようになる。

$$m\ddot{y}=-c_l(\dot{y}-\dot{y}_c)-c_f\dot{y}-k(y-y_c)-k_sy-s_1 \tag{1}$$

$$\ddot{y}=-\frac{c_l}{m}(\dot{y}-\dot{y}_c)-\frac{c_f}{m}\dot{y}-\frac{k}{m}(y-y_c)-\frac{k_s}{m}y-\frac{s_1}{m} \tag{2}$$

いま,

$$\frac{c_l}{m}=a,\quad \frac{c_f}{m}=b,\quad \frac{k}{m}=c,\quad \frac{k_s}{m}=d,\quad \frac{s_1}{m}=e$$

とおくと

$$\ddot{y}+a(\dot{y}-\dot{y}_c)+b\dot{y}+cy+dy=cy_c-e \tag{3}$$

$$\therefore\quad \ddot{y}+(a+b)\dot{y}+(c+d)y=a\dot{y}_c+cy_c-e \tag{4}$$

正弦曲線運動として零より出発して弁上昇行程，$h$cm に達する時の時間を t_0 とすると，カムの変位は次のようになる。

$$y_c=\frac{h}{2}\left(1-\cos\frac{t}{t_0}\pi\right)$$

カム速度は

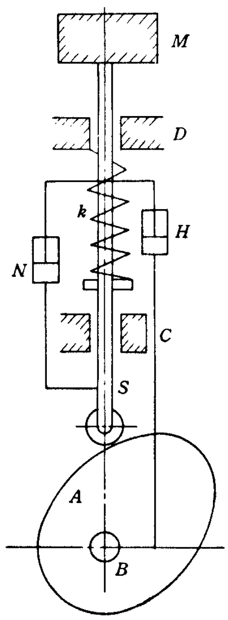
$$\dot{y}_c=\frac{h}{2}\ \frac{\pi}{t_0}\sin\frac{\pi t}{t_0}\qquad 0\leqq t\leqq t_0$$

カム加速度は

$$\ddot{y}=\frac{h}{2}\ \frac{\pi^2}{t_0{}^2}\cos\frac{\pi}{t_0}$$

これを（4）式に代入して

$$\ddot{y}=(a+b)\dot{y}+(c+d)y=a\frac{h}{2}\ \frac{\pi}{t_0}\sin\frac{\pi t}{t_0}+\frac{ch}{2}\left(1-\cos\frac{\pi t}{t_0}\right)-e \tag{5}$$



A=カム
B=カム軸
s=従動節
k=スプリング
N,H=減衰箱
M=質量
C,D=支持物

第1図　カ　ム　系

なお,

$$\frac{a}{2}\ \frac{\pi}{t_0}=\alpha,\qquad \frac{ch}{2}=\beta\ \text{と置くと}$$

$$\ddot{y}+(a+b)\dot{y}+(c+d)y=\alpha\sin\frac{\pi t}{t_0}-\beta\cos\frac{\pi t}{t_0}+\beta-e \tag{6}$$

両辺を Laplace 変換して虚像を求めると

$$s^2Y(s)-s\cdot y(0)-y'(0)+(a+b\{s\cdot Y(s)-y(0)\}$$
$$+(c+d)Y(s)=\alpha\frac{\left(\frac{\pi}{t_0}\right)}{s^2+\left(\frac{\pi}{t_0}\right)^2}-\beta\frac{s}{s^2+\left(\frac{\pi}{t_0}\right)^2}+\frac{(\beta-e)}{s} \tag{7}$$

しかし正弦曲線カムにおいては $t=0$ における変位および速度は零であるから

$$t=0 \longrightarrow y(0)=0$$

$$t=0 \longrightarrow y'(0)=0$$

したがって

$$\therefore s^2 \cdot Y(s)+(a+b)s \cdot Y(s)+(c+d) \cdot Y(s)=\frac{\alpha\left(\frac{\pi}{t_0}\right)}{s^2+\left(\frac{\pi}{t_0}\right)^2}$$

$$-\beta \cdot \frac{s}{s^2+\left(\frac{\pi}{t_0}\right)^2}+\frac{(\beta-e)}{s} \tag{7}$$

$$\therefore Y(s)=\frac{1}{s^2+(a+b)s+(c+d)}\left[\frac{\alpha\left(\frac{\pi}{t_0}\right)}{s^2+\left(\frac{\pi}{t_0}\right)^2}-\beta\frac{s}{s^2+\left(\frac{\pi}{t_0}\right)^2}+\frac{\beta-e}{s}\right] \tag{8}$$

いま $\frac{\pi}{t_0}=p$ とおくと

$$Y(s)=\frac{1}{s^2+(a+b)s+(c+a)}\left[\frac{\alpha p}{s^2+p^2}-\beta\frac{s}{s^2+p^2}+\frac{\beta-e}{s}\right] \tag{9}$$

なお，$a+b=a_1$，$c+d=b_1$ と置くと

$$Y(s)=\frac{1}{s^2+a_1s+b_1}\left[\frac{\alpha p}{s^2+p^2}-\beta\frac{s}{s^2+p^2}+\frac{\beta-e}{s}\right] \tag{10}$$

ここに

$$\frac{\alpha}{(s^2+a_1s+b_1)(s^2+p^2)}=\frac{A_1s+B_1}{(s^2+a_1s+b_1)}+\frac{c_1s+D_1}{s^2+p^2} \quad \text{であって}$$

$$A_1=\frac{a_1\alpha}{\{a_1^2b_1-(a_1^2-b_1+p^2)(p^2-b_1)\}}, \qquad p=\frac{\pi}{t_0}, \quad a_1=\frac{c_l+c_f}{m}, \quad b_1=\frac{k+k_s}{m}, \quad \alpha=\frac{c_l}{m}\frac{h}{2}\frac{\pi}{t_0}$$

$$\therefore A_1=\left(\frac{c_l+c_f}{m}\right)\frac{\pi hc_l}{2mt_0}\Big/\left[\left(\frac{c_l+c_f}{m}\right)^2\left(\frac{k+k_s}{m}\right)-\left\{\left(\frac{c_l+c_f}{m}\right)^2-\left(\frac{k+k_s}{m}\right)^2\right.\right.$$

$$\left.\left.+\left(\frac{\pi}{t_0}\right)^2\right\}\left\{\left(\frac{\pi}{t_0}\right)^2-\left(\frac{k+k_s}{m}\right)\right\}\right]$$

$$B_1=\frac{\alpha(p-b_1)+a_1^2\alpha}{\{a_1^2b_1-(a_1^2-b_1^2+p^2)(p^2-b_1)\}}$$

$$=\left\{\frac{\pi hc_l}{2mt_0}\left(\frac{\pi}{t_0}-\frac{k+k_s}{m}\right)+\left(\frac{c_l+c_f}{m}\right)^2\left(\frac{\pi hc_l}{2mt_0}\right)\right\}\Big/\left[\left(\frac{c_l+c_f}{m}\right)^2\left(\frac{k+k_s}{m}\right)\right.$$

$$\left.-\left\{\left(\frac{c_l+c_f}{m}\right)^2-\left(\frac{k+k_s}{m}\right)^2+\left(\frac{\pi}{t_0}\right)^2\right\}\left\{\left(\frac{\pi}{t_0}\right)^2-\left(\frac{k+k_s}{m}\right)\right\}\right]$$

$$C_1=-\frac{a_1\alpha}{\{\quad〃\quad\}}=-\left(\frac{c_l+c_f}{m}\right)\frac{\pi hc_l}{2mt_0}\Big/\Big[\qquad〃\qquad\Big]$$

$$D_1=\frac{-\alpha(p^2-b_1)}{\{\quad〃\quad\}}=-\frac{\pi hc_l}{2mt_0}\Big/\Big[\qquad〃\qquad\Big]$$

次の虚像の値を求めると

$$\frac{s}{\{s^2+(a+b)s+(c+d)\}(s^2+p^2)}=\frac{As}{s_2+a_1s+b_1}+\frac{B}{s^2+a_1s+b_1}$$

$$+\frac{Cs}{s_2+p^2}+\frac{D}{s^2+p^2}$$

ここに

$$A=\frac{(p^2-b_1)}{\{a_1{}^2b_1+(a_1{}^2-b_1+p^2)(p^2-b_1)\}}=\left(\frac{\pi}{t_0}-\frac{k+k_s}{m}\right)\Big/\Big[\left(\frac{c_l+c_f}{m}\right)^2$$

$$\cdot\left(\frac{k+k_s}{m}\right)-\left\{\left(\frac{c_l+c_f}{m}\right)^2-\left(\frac{k+k_s}{m}\right)^2+\frac{\pi}{t_0}\right\}\left\{\frac{\pi}{t_0}-\frac{k+k_s}{m}\right\}\Big]$$

$$B=-\frac{1}{a_1}-\frac{\left(p^2-b_1\right)\left(-\frac{b_1}{a_1}+\frac{p^2}{a_1}+a_1\right)}{\{a_1{}^2b_1(a_1{}^2-b_1+p^2)(p^2-b_1)\}}$$

$$=\left\{-1\Big/\left(\frac{c_l+c_f}{m}\right)\right\}-\left(\frac{\pi}{t_0}-\frac{k+k_s}{m}\right)\left\{-\frac{(k+k_s)}{c_l+c_f}+\frac{\pi m}{t_0(c_l+c_f)}+\frac{(c_l+c_f)}{m}\right\}\Big/$$

$$\Big[\left(\frac{c_l+c_f}{m}\right)^2\left(\frac{k+k_s}{m}\right)-\left\{\left(\frac{c_l+c_f}{m}\right)^2-\left(\frac{k+k_s}{m}\right)^2+\frac{\pi}{t_0}\right\}\left\{\frac{\pi}{t_0}-\frac{k+k_s}{m}\right\}\Big]$$

$$C=\frac{-(p^2-b_1)}{\{a_1{}^2b_1+(a_1{}^2-b_1+p^2)(p^2-b_1)\}}=-\left(\frac{\pi}{t_0}-\frac{k+k_s}{m}\right)\Big/\Big[\qquad〃\qquad\Big]$$

$$D=\frac{1}{a_1}\left[1-\frac{(p^2-b_1)^2}{\{a_1{}^2b_1+(a_1{}^2-b_1+p^2)(p^2-b_1)\}}\right]=\frac{m}{(c_l+c_f)}$$

$$\cdot\left[1-\left(\frac{\pi}{t_0}-\frac{k+k_s}{m}\right)^2\Big/\Big[\qquad〃\qquad\Big]\right.$$

また

$$\frac{1}{(s^2+a_1s+b_1)s}=\frac{A_2s}{s^2+a_1s+b_1}+\frac{B_2}{s^2+a_1s+b_1}+\frac{C_2}{s}$$

ここに

$$A_2=-\frac{1}{b_1}=\frac{-m}{(k+k_s)}$$

$$B_2=-\frac{a_1}{b_1}=-(c_l+c_f)/(k+ks)$$

$$C_2=\frac{1}{b_1}=\frac{m}{(k+k_s)}$$

したがって上式を整頓すると次のようになる。

$$\therefore Y(s)=\frac{1}{(s^2+a_1s+b_1)}\left[\frac{\alpha p^2}{s^2+p^2}-\beta\frac{s}{s^2+p^2}+\frac{\beta-e}{s}\right]$$

$$=p^2\left(\frac{A_1s+B_1}{s^2+a_1s+b_1}+\frac{C_1s+D_1}{s^2+p^2}\right)-\beta\left(\frac{As+B}{s^2+a_1s+b_1}+\frac{Cs+D}{s^2+p^2}\right)$$

$$+(\beta-e)\left(\frac{As_2+B_2}{s^2+a_1s+b_1}+\frac{C_2}{s}\right)=\frac{\{p^2A_1-\beta A+(\beta-e)A_2\}s}{s^2+a_1s+b_1}$$

$$+\frac{\{B_1p^2-\beta B+(\beta-e)B_2\}}{s^2+a_1s+b_1}+\frac{(p^2C_1-\beta C)s}{s^2+p^2}+\frac{p^2D_1-\beta D}{s^2+p^2}+\frac{(\beta-e)C_2}{s}$$

$$=\{p^2A_1-\beta A+(\beta-e)A_2\}\frac{\left(s+\frac{a_1}{2}\right)-\frac{a_1}{2}}{\left(s+\frac{a_1}{2}\right)^2+\left(\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\right)^2}+\frac{\{B_1p^2-\beta B+(\beta-e)B_2\}}{\left(s+\frac{a_1{}^2}{2}\right)^2+\left(\sqrt{b_1{}^2-\left(\frac{a_1}{4}\right)^2}\right)^2}$$

$$+\frac{(p^2C_1-\beta C)s}{s^2+p^2}\cdot\frac{s}{s^2+p^2}+(p^2D_1-\beta D)\frac{1}{p}\cdot\frac{p}{s^2+p^2}+(\beta-e)C_2\cdot\frac{1}{s}$$

とし

$$p^2A_1-\beta A+(\beta-e)A_2=\xi_1\ ;\ \{B_1p^2-\beta B+(\beta-eB\}=\xi_2\ ;$$

$$p^2C_1-\beta C=\eta_1\ ;\ (p^2D_1-\beta D)=\eta_2\ ;\ (p-e)C_2=\eta_3$$

とおくと

$$Y(s)=\xi_1\frac{s+\frac{a_1}{2}-\frac{a_1}{2}}{\left(s+\frac{a_1}{2}\right)^2+\left(\sqrt{b_1{}^2-\frac{a_1}{4}}\right)^2}+\xi_2\frac{1}{\left(s+\frac{a_1}{2}\right)^2+\left(\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\right)^2}$$

$$+\eta_1\frac{s}{s^2+p^2}+\eta_2\frac{1}{p}\cdot\frac{p}{s_2+p^2}+\eta_3\frac{1}{s}$$

この虚関数を実関数に戻すと変位 y を求めることが出来，その値は次のようになる。

$$y=\xi_1e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t-\xi_1\frac{a_1}{2}e^{-\frac{a_1}{2}t}\frac{1}{\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$+\xi_2\frac{1}{\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}}e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t+\eta_1\cos pt+\eta_2\frac{1}{p}\sin pt+\eta_3$$

$$\therefore y=\xi_1 e^{-\frac{1}{2}\left(\frac{c_l+c_f}{m}\right)t}\cdot\cos\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}\cdot t$$

$$-\xi_1\frac{(c_l+c_f)}{2m}e^{-\frac{(c_l+c_f)}{2m}t}\frac{1}{\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}}\cdot\sin\sqrt{\left(\frac{+\ !\ {}_x}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}$$

$$\cdot t+\xi_2\frac{1}{\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}}e^{-\frac{(c_l+c_f)}{2m}\cdot t}\cdot\sin\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}\cdot t$$

$$+\eta_1\frac{t_0}{\pi}\sin\frac{\pi}{t_0}t+\eta_3 \qquad (11)$$

III.　速度曲線，加速度曲線，パルス曲線方程式

次に速度曲線を求めると

$$\dot{y}=\xi_1\Big[e^{-\frac{a}{2}t}\left(-\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\sin\sqrt{d_1{}^2-\frac{a_1{}^2}{4}}\cdot t\right)-\frac{a_1}{2}e^{-\frac{a_1}{2}t}\cdot\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$+\left(-\xi_1\frac{a_1}{2}+\frac{\xi_2}{\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}}\right)\Big]\Big[\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\,e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$-\frac{a_1}{2}e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\Big]-\eta_1 p\sin pt+\eta_2\cos pt$$

$$=\left\{-\xi_1\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}+\left(\xi_1\frac{a_1{}^2}{4}-\frac{\xi_2 a_1}{2}\frac{1}{\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}}\right)\right\}\cdot e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a^2}{4}}\cdot t$$

$$+\left\{-\xi_1\frac{a_1}{2}\left(1+\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\right)+\xi_2\right\}e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$-\eta_1 p\sin pt+\eta_2\cos pt \qquad (12)$$

前式において

$$\left\{-\xi_1\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}+\left(\xi_1\frac{a_1{}^2}{4}-\xi_2\frac{a_1}{2}\frac{1}{\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}}\right)\right\}=\Phi_1$$

$$\left\{-\xi_1\frac{a_1}{2}\left(1+\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\right)+\xi_2\right\}=\Phi_2 \qquad \text{と置けば}$$

速度方程式は次のようになる。

$$\dot{y}=\Phi_1 e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t+\Phi_2 e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$-\eta_1 p\sin pt+\eta_2\cos pt \tag{13}$$

次に加速度曲線方程式は

$$\ddot{y}=\left[-\frac{a_1}{2}\Phi_1-\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot\Phi_2\right]e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t$$

$$+\left[\Phi_1\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}-\frac{a_1}{2}\Phi_2\right]e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\cdot t-\eta_1 p^2\cos pt-\eta_2 p\sin pt.$$

$$=\left[-\frac{c_l+c_f}{2m}\left\{-\xi_1\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}+\xi_1\frac{(c_l+c_f)^2}{4}-\xi_2\frac{(c_l+c_f)}{2m}\cdot\right.\right.$$

$$\left.\frac{1}{\sqrt{\left(\frac{k+k_x}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}}\right\}-\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}\left\{-\xi_1\frac{(c_l+c_f)}{2m}\right.$$

$$\left.\left.-\xi_1\frac{(c_l+c_f)}{2m}\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}+\xi_3\right\}\cdot e^{-\frac{(c_l+c_f)}{2m}t}\cdot\right.$$

$$\sin\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)}\cdot t+\left[-\xi_1\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}\right.$$

$$\left.+\xi_1\frac{(c_l+c_f)}{4}-\xi_2\frac{(c_l+c_f)}{2m}\quad\frac{1}{\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}}\right\}\cdot\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}$$

$$\left.+\frac{(c_l+c_f)}{2m}\left\{\xi_1\left(\frac{c_l+c_f}{2m}\right)+\xi_1\frac{(c_l+c_f)}{2m}\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}+\xi_3\right\}\cdot\right.$$

$$e^{-\frac{(c_l+c_f)}{2m}t}\cos\sqrt{\left(\frac{k+k_s}{m}\right)^2-\frac{1}{4}\left(\frac{c_l+c_f}{m}\right)^2}\cdot t$$

$$-\eta_1\left(\frac{\pi}{t_0}\right)^2\cos\frac{\pi}{t_0}t-\eta_2\frac{\pi}{t_0}\sin\frac{\pi}{t_0}t \tag{14}$$

更にパルス曲線方程式を求めると次のようになる。

$$\dddot{y}=A\left[\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\,t-\frac{a_1}{2}e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\,t\right]$$

$$+B\left[-\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}e^{-\frac{a_1}{2}t}\sin\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\,t-\frac{a_1}{2}e^{-\frac{a_1}{2}t}\cos\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\,t\right]$$

$$+\eta_1 p^3\sin pt-\eta_2 p^2\cos pt \tag{15}$$

ここに

$$A=\left[-\frac{a_1}{2}\Phi_1-\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}\Phi_2\right]$$

$$B=\left[\Phi_1\sqrt{b_1{}^2-\frac{a_1{}^2}{4}}-\frac{a_1}{2}\Phi_2\right]$$

IV. 回転摩擦のない場合

この場合は $c_f\dot{y}$ の項が消滅することになるから振動微分方程式は

$$\ddot{y}+\left(\frac{k+k_s}{m}\right)y=\frac{k}{m}y_c-\frac{s_1}{m} \tag{16}$$

前と同様

$$\frac{k}{m}=c,\quad \frac{k_s}{m}=d\quad \frac{s_1}{m}=e$$ と置くと

$$\ddot{y}+(c+d)y=cy_c-e \tag{17}$$

カム変位を

$$y_c=\frac{h}{2}\left(1-\cos\frac{\pi}{t_0}t\right)=\frac{h}{2}(1-\cos\omega t),\quad \omega=\frac{\pi}{t_0}\quad 0\leqq t\leqq t_0$$

とすると

$$\ddot{y}+(c+d)y=cy_c-e=\frac{ch}{2}(1-\cos\omega t)-e \tag{18}$$

$(c+d)=a^2$ として両辺にラプラス変換をすると

$$s^2Y(s)-sy(0)-y'(0)+a^2Y(s)=\frac{ch}{2}\left[\frac{1}{s}-\frac{\omega}{s^2+\omega^2}\right]-\frac{e}{s}$$

$$=\left(\frac{ch}{2}-e\right)\frac{1}{s}-\frac{ch\omega}{2}\cdot\frac{1}{s^2+\omega^2} \tag{19}$$

初期条件として

$$t=0 \longrightarrow y(0)=0$$

$$t=0 \longrightarrow y'(0)=0$$

$$\therefore s^2 Y(s)+a^2 Y(s)=\left(\frac{ch}{2}-e\right)\frac{1}{s}-\frac{ch\omega}{2}\cdot\frac{1}{s^2+\omega^2}$$

$$Y(s)\left[s^2+a^2\right]=\left(\frac{ch}{2}-e\right)\frac{1}{s}-\frac{ch\omega}{2}\ \frac{1}{s^2+\omega^2}$$

故に

$$Y(s)=\left(\frac{ch-2e}{2}\right)\frac{1}{s(s^2+a^2)}-\frac{chw}{2}\ \frac{1}{(s^2+a^2)(s^2+\omega^2)}$$

$$=\left(\frac{ch-2e}{e}\right)\left[\frac{1}{a^2}\left(\frac{1}{s}-\frac{s}{s^2+a^2}\right)\right]-\frac{ch\omega}{2}\left[\frac{1}{(a^2-\omega^2)}\left\{\frac{1}{s^2+\omega_2}-\frac{1}{s^2+a^2}\right\}\right]$$

$$\therefore y=\left(\frac{ch-2e}{2}\right)\frac{1}{a^2}\left[1-\cos\ at\right]-\frac{ch\omega}{2(a^2-\omega^2)}\left[\frac{1}{\omega}\sin\omega t-\frac{1}{a}\sin\ at\right]$$

$$=\frac{(kh-2s)}{2(k+k_s)}\left[1-\cos\sqrt{\frac{k+k_s}{m}}\cdot t\right]$$

$$-\frac{kh\omega t_0{}^2}{2\{t_0{}^2(k+k_s)-m\pi^2\}}\left[\frac{t_0}{\omega}\sin\frac{\omega}{t_0}\cdot t-\sqrt{\frac{m}{k+k_s}}\sin\sqrt{\frac{k+k_s}{m}}\cdot t\right]$$

したがって

$$\dot{y}=\left(\frac{kh-2s_1}{m}\right)\sqrt{\frac{m}{k+k_s}}\cdot\sin\sqrt{\frac{k+k_s}{m}}\cdot t$$

$$-\frac{kh\pi t_0}{2\{t_0{}^2(k+k_s)-m\pi^2 t\}}\left[\cos\frac{\pi}{t_0}t-\cos\sqrt{\frac{k+k_s}{m}}\cdot t\right]$$

加速度は

$$\ddot{y}=\frac{kh-2s_1}{2m}\cos\sqrt{\frac{k+k_s}{m}}\cdot t-\frac{kh\pi t_0}{2\{t_0{}^2(k+k_s)-m\pi^2\}}$$

$$\left[\sqrt{\frac{k+k_s}{m}}\sin\sqrt{\frac{k+k_s}{m}}t-\frac{\pi}{t_0}\sin\frac{\pi}{t_0}t\right]$$

パルスは

$$\dddot{y}=-\left(\frac{kh-2s_1}{2m}\right)\sqrt{\frac{k+k_s}{m}}\sin\sqrt{\frac{k+k_s}{m}}\cdot t$$

$$\frac{-kh\pi t_0}{2\{t_0(k+k_s)-m\pi^2\}}\left[\sqrt{\frac{k+k_s}{m}}\cdot\frac{\pi}{t_0}\cos\sqrt{\frac{k+k_s}{m}}\cdot t-\frac{\pi^2}{t_0{}^2}\cos\frac{\pi}{t_0}\ t\right]$$

となる。

(16) 計　算　例

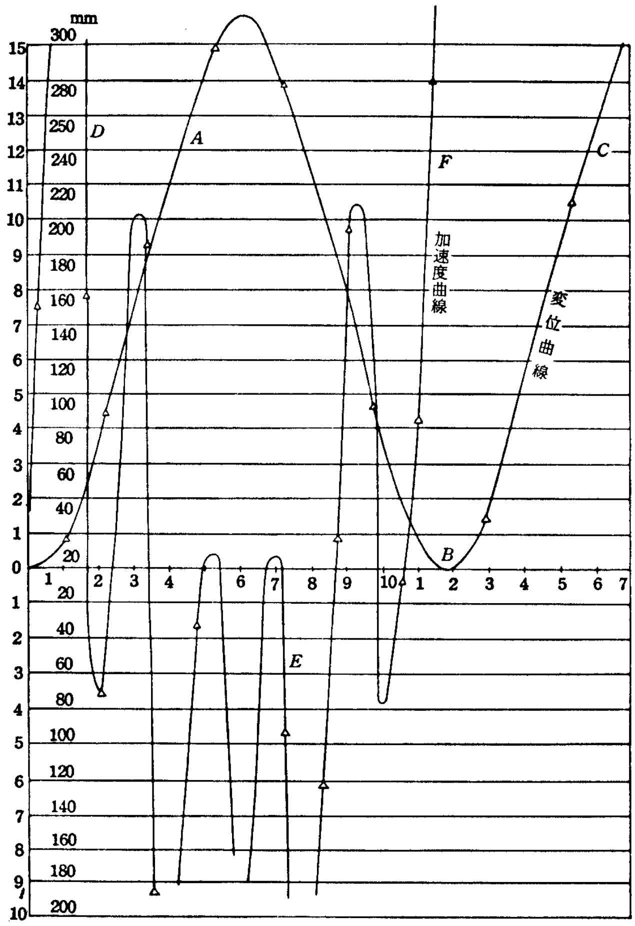
カム振動系の例として

（A）　$72\times10^{-2}\ddot{y}+0.003\dot{y}+2y=16\ \{1-0.9\cos10\pi t\}$　と

（B）　$72\times10^{-2}\ddot{y}+0.02\dot{y}+2y=16\{1-0.9\cos10\pi t\}$

とを計算してみると次表のようになる。

また x 軸に時間をとり y 軸に変位および加速度をとり曲線を描けば次図のようになる。



第2図　変位曲線・加速度曲線

(A) t sec	y	$\dot{y}$	$\ddot{y}$	$\dddot{y}$
0.0000	0.00000	0.00000	22.22232	-0.22593
0.0200	0.00574	0.70128	60.23032	3690.55762
0.0400	0.03764	2.82164	152.25613	5968.60156
0.0600	0.13399	7.21848	280.00342	5962.80469
0.0800	0.34143	13.84103	373.96509	3675.17836
0.1000	0.69622	21.73172	401.97778	-20.73648
0.1200	1.20905	29.37459	349.21704	-3713.41187
0.1400	1.85933	35.25769	230.90796	-5992.58203
0.1600	2.60110	38.43510	86.56380	-5987.86719
0.1800	3.37826	38.87395	-35.04199	-3701.22730
0.2000	4.14355	37.45386	-94.43634	-6.25813
0.2200	4.87365	35.61948	-76.44370	3685.58936
0.2400	5.57533	34.81883	4.09915	5964.03906
0.2600	6.27944	35.94199	108.10083	5958.69922
0.2800	7.02594	38.97487	187.23755	3671.56274
0.3000	7.84517	43.00000	202.50952	-23.79361
0.3200	8.74273	46.54753	139.23169	-3715.86255
0.3400	9.69384	48.14215	12.44463	-5994.41016
0.3600	10.64983	46.89448	-134.34912	-5989.03516
0.3800	11.55055	42.81859	-262.21533	-3701.72559
0.4000	12.34918	36.84344	-322.34570	-4.07906
0.4200	13.02159	30.46458	-302.55981	3686.41113
0.4400	13.57519	25.18048	-217.41631	5965.48047
0.4600	14.04258	21.93092	-106.95728	5960.75000
0.4800	14.46643	20.75020	-18.68188	3674.20117
0.5000	14.88070	20.76743	8.00415	-20.59977
0.5200	15.29558	20.55406	-41.41045	-3712.19971
0.5400	15.69169	18.68536	-152.63590	-5990.15234
0.5600	16.02600	14.30131	-285.34441	-5984.06172
0.5800	16.24706	7.45275	-391.20020	-3696.53735
0.6000	16.31389	-0.90003	-430.90454	-0.50669
0.6200	16.21117	-9.23573	-389.50513	3692.20093
0.6400	15.95490	-16.03455	-282.10107	5971.46094
0.6600	15.58665	-20.34021	-178.08740	5966.81641
0.6800	15.15843	-22.10712	-36.11865	3680.28467
0.7000	14.71366	-22.20003	14.43433	-14.45831
0.7200	14.27207	-22.04671	-11.50488	-3706.07202
0.7400	13.82378	-23.07631	-99.00977	-5984.15625
0.7600	13.33513	-26.15855	-208.91528	-5978.45703
0.7800	12.76379	-31.25740	-292.83032	-3690.28462
0.8000	12.07453	-37.43243	-311.70554	4.72852
0.8200	11.26930	-43.18680	-250.81665	3697.05981
0.8400	10.36294	-47.02969	-125.47754	5975.89063
0.8600	9.40723	-48.03801	24.71179	5970.76953
0.8800	8.46059	-46.20300	150.93677	3683.41384
0.9000	7.57217	-42.42961	213.65234	-11.53829
0.9200	6.76653	-38.18871	197.65527	-3703.79272
0.9400	6.03478	-34.95509	117.46318	-5980.49219
0.9600	5.35529	-33.64586	12.05899	-5977.42578
0.9800	4.67883	-34.27379	-68.12360	-3690.52393
1.0000	3.97724	-35.94809	-86.75902	4.46489
1.0200	3.24358	-37.22205	-28.13803	3695.68481
1.0400	2.50077	-36.65492	92.44371	5973.04141
1.0600	1.79570	-33.34250	235.47908	5969.20703
1.0800	1.18393	-27.41316	352.23486	3683.05688
1.1000	0.71082	-19.72678	403.29956	-11.64178
1.1200	0.39628	-11.82678	373.58179	-3704.44946
1.1400	0.22894	-5.22814	278.01782	-5985.41016
1.1600	0.17191	-0.88565	155.83682	-5983.81250
1.1800	0.17795	1.15332	55.53612	-3700.45679
1.2000	0.20829	1.44895	16.36372	-4.49924
1.2200	0.24776	2.32229	53.30410	3684.62573
1.2400	0.31063	4.29439	151.37430	5966.01353

1.2600	0.43480	8.52545	271.33667	5965.22266
1.2800	0.66661	14.96850	364.71484	3682.30908
1.2999	1.04206	22.66995	392.33154	-11.22173
1.3199	1.57171	30.11710	339.30688	-3705.12769
1.3399	2.23485	35.79987	220.78906	-5989.26953
1.3599	2.98542	38.77281	76.22844	-5991.50781
1.3799	3.76726	39.00256	-45.62003	-3711.23926
1.3999	4.53296	37.36847	-105.24995	-19.21211
1.4199	5.25920	35.31616	-87.39619	3674.62891
1.4399	5.95262	34.29665	-6.79500	5959.68750
1.4599	6.64413	35.20482	97.54614	5963.42188
1.4799	7.37382	38.03250	177.32056	3684.30103
1.4999	8.17227	41.86775	193.46997	-4.32633
1.5199	9.04544	45.24141	131.19482	-3701.98779
1.5399	9.96894	46.68832	5.69629	-5988.76953
1.5599	10.89392	45.30898	-143.52417	-5994.43750
1.5799	11.76293	41.11643	-267.67139	-3716.76880
1.5999	12.52588	35.03876	-327.1[illegible]	-25.43619
1.6199	13.16127	28.56961	-306.[illegible]	3670.29370
1.6399	13.67619	23.20972	-221.0[illegible]	5959.30469
1.6599	14.10356	19.90343	-109.[illegible]	5967.69922
1.6799	14.48649	18.69073	-19.[illegible]202	3692.47607
1.6999	14.85950	18.70560	8.[illegible]773	2.56233
1.7199	15.23338	18.52164	-39.45630	-3692.74561
1.7399	15.58939	16.71246	-148.97827	-5982.36719
1.7599	15.88506	12.41302	-280.60156	-5991.67969
1.7799	16.06938	5.66734	-385.71191	-3716.97559
1.7999	16.10165	-2.57058	-424.93750	-26.52214
1.8199	15.96675	-10.78309	-383.15039	3670.76831
1.8399	15.68080	-17.45024	-275.26074	5963.13672
1.8599	15.28566	-21.61229	-140.52441	5975.66016
1.8799	14.83356	-23.21861	-27.58813	3703.69360
1.8998	14.36835	-23.13000	24.06274	15.10612
1.9198	13.91015	-22.77361	-0.85986	-3681.59253
1.9398	13.44951	-23.58260	-87.66406	-5974.57422
1.9598	12.95302	-26.43472	-197.33691	-5988.10156
1.9798	12.37846	-31.30397	-281.52344	-3716.86255
1.9998	11.69350	-37.25883	-301.04248	-28.07657
2.0198	10.89082	-42.80780	-240.94092	3669.98486
2.0398	9.99396	-46.46034	-116.22185	5965.24609
2.0598	9.05145	-47.28923	33.51758	5981.33594
2.0798	8.12154	-45.27953	159.65210	3712.17041
2.0998	7.25333	-41.33102	222.45996	24.20697
2.1198	6.47143	-36.91341	206.48364	-3674.23340
2.1398	5.76994	-33.50566	126.27100	-5971.11719
2.1598	5.11809	-32.03366	20.73131	-5989.33594
2.1798	4.47532	-32.52098	-61.81602	-3722.07178

(B)　t sec	y	$\dot{y}$	$\ddot{y}$	$\dddot{y}$
0.0000	0.00000	0.00000	22.22232	-6.14287
0.0200	0.00573	0.69993	60.06534	3676.22321
0.0400	0.03755	2.81280	158.50497	5930.72656
0.0600	0.13353	7.18713	278.32056	5895.75391
0.0800	0.33991	13.76150	370.76144	3584.59619
0.1000	0.69235	21.57050	396.99878	-190.37568
0.1200	1.20084	29.09673	372.58642	-3803.09414
0.1400	1.84416	34.83459	223.12231	-6059.50781
0.1600	2.57586	37.85075	78.35213	-6025.56250
0.1800	3.33971	38.12737	-42.94220	-3715.96660
0.2000	4.08851	36.55699	-101.50224	-11.22296
0.2200	4.79935	34.59138	-82.50436	3671.64868
0.2400	5.47931	33.67830	-1.13811	5926.52344
0.2600	6.15960	34.70169	103.98784	5895.03306
0.2800	6.88035	37.63910	182.45068	3581.32837
0.3000	7.67189	41.56629	197.56812	-123.00269
0.3200	8.53980	45.01254	134.30444	-3806.00977
0.3400	9.45930	46.51552	8.34473	-6060.89844
0.3600	10.38135	45.19131	-140.50708	-6026.29687
0.3800	11.24809	41.07031	-263.43774	-3715.34253
0.4000	12.01166	35.09410	-321.18359	-10.65402
0.4200	12.64948	28.76324	-298.94409	3672.83008
0.4400	13.16993	23.57364	-211.95825	5928.29297
0.4600	13.60645	20.45613	-99.61133	5894.36719
0.4800	14.00234	19.43330	-10.32397	3584.19702
0.5000	14.39200	19.62436	16.99341	-119.62053
0.5200	14.73585	19.59637	-32.13843	-3802.22900
0.5400	15.16478	17.92706	-142.20093	-6056.58203
0.5600	15.48609	13.76425	-273.57739	-6021.53516
0.5800	15.69891	7.16825	-377.65430	-3710.22827
0.6000	15.76292	-0.89395	-415.38867	-5.22217
0.6200	15.66352	-8.90084	-372.20508	3678.41309
0.6400	15.41744	-15.34026	-263.57983	5934.00000
0.6600	15.06686	-19.26985	-129.14355	5900.10156
0.6800	14.66382	-20.66051	-17.56128	3589.88916
0.7000	14.25164	-20.39142	32.00732	-113.93636
0.7200	13.84966	-19.89885	4.83765	-3796.59521
0.7400	13.44751	-20.61327	-83.79346	-6051.12109
0.7600	13.01110	-23.39983	-194.50464	-6016.30078
0.7800	12.49778	-28.21544	-278.90576	-3705.27539
0.8000	11.87512	-34.11604	-298.16626	-0.60699
0.8200	11.13589	-39.60522	-237.90869	3682.64111
0.8400	10.30367	-43.20096	-113.70053	5937.79688
0.8600	9.42675	-43.99425	34.39429	5903.42578
0.8800	8.56275	-41.99232	157.83838	3592.70850
0.9000	7.75971	-38.11327	217.26196	-111.65117
0.9200	7.04089	-33.83453	197.84155	-3794.36646
0.9400	6.39904	-30.62958	114.78003	-6049.96484
0.9600	5.80228	-29.40759	7.40965	-6015.72656
0.9800	5.20930	-30.17061	-45.00630	-3705.21167
1.0000	4.58809	-32.01880	-96.33087	-1.19964
1.0200	3.93099	-33.50165	-39.47856	3681.01196
1.0400	3.26019	-33.18120	79.03841	5935.67187
1.0600	2.62175	-30.19144	219.53003	5901.78516
1.0800	2.07022	-24.58038	333.30127	3592.05420
1.1000	1.64944	-17.30431	381.20239	-111.62406
1.1200	1.37906	-9.87646	374.53296	-3795.37280
1.1400	1.24555	-3.80358	250.61743	-6052.66406
1.1600	1.21141	-0.02565	126.93192	-6021.20078
1.1800	1.22831	1.42754	25.99695	-3714.44023
1.2000	1.25879	1.43188	-13.13983	-12.64286
1.2200	1.28197	1.41858	24.17061	3670.57178
1.2400	1.32497	2.81178	122.59983	5928.46484

1.2600	1.41374	6.46913	242.70215	5898.57031
1.2800	1.59869	12.33910	336.02002	3592.11841
1.2999	1.91581	19.46597	363.59814	-110.37376
1.3199	2.37564	26.34090	310.91333	-3795.20679
1.3399	2.95764	31.46503	193.46313	-6055.66797
1.3599	3.61617	33.90985	50.90302	-6028.62891
1.3799	4.29586	33.66090	-68.02850	-3724.62744
1.3999	4.95048	31.61368	-124.07217	-23.44731
1.4199	5.55812	29.22353	-102.34546	3661.50415
1.4399	6.12695	27.94319	-17.97037	5923.05078
1.4599	6.68038	28.66280	89.79395	5897.62844
1.4799	7.28689	31.36636	172.60693	3594.91699
1.4999	7.95127	35.13580	191.50351	-105.76556
1.5199	8.68962	38.50096	132.26904	-3791.45801
1.5399	9.47844	40.00221	10.14673	-6054.66016
1.5599	10.27114	38.75056	-135.10083	-6031.14062
1.5799	11.01034	34.77122	-254.72876	-3729.82129
1.5999	11.64991	28.99991	-309.44098	-29.41159
1.6199	12.16839	22.93271	-284.34009	3657.34473
1.6399	12.57533	18.06329	-194.54468	5922.74609
1.6599	12.90531	15.32230	-79.36499	5901.89453
1.6799	13.20277	14.73292	12.77295	3602.01650
1.6999	13.50322	15.41382	42.84961	-96.19775
1.7199	13.81821	15.92848	-3.79883	-3782.68091
1.7399	14.12958	14.84684	-111.85596	-6048.85547
1.7599	14.39544	11.30584	-241.84717	-6029.02984
1.7799	14.56552	5.35285	-345.18872	-3730.21934
1.7999	14.59974	-2.05753	-382.75317	-21.35680
1.8199	14.48359	-9.41341	-339.78589	3656.90430
1.8399	14.23374	-15.20833	-231.56714	5925.60156
1.8599	13.89215	-18.50211	-97.57666	5908.20859
1.8799	13.51074	-19.26602	13.54126	3613.02271
1.8998	13.13964	-18.38034	62.52979	-84.23676
1.9198	12.77692	-17.28551	34.47144	-3742.76880
1.9398	12.43289	-17.42035	-55.56819	-6042.32203
1.9598	12.06586	-19.66182	-168.32350	-6026.72195
1.9798	11.63237	-23.98039	-255.44141	-3731.27219
1.9998	11.09889	-29.44275	-277.80941	-24.02798
2.0198	10.45696	-34.56078	-221.07251	3655.01733
2.0398	9.72877	-37.85390	-100.33545	5926.68750
2.0598	8.96127	-38.40988	44.42168	5913.53195
2.0798	8.21082	-36.23006	165.37937	3620.61450
2.0998	7.52435	-32.22708	222.16138	-76.54976
2.1198	6.92406	-27.87637	200.11597	-3766.14160
2.1398	6.40165	-24.65411	114.17578	-6039.50781
2.1598	5.92406	-23.47624	3.52974	-6028.51953
2.1798	5.44860	-24.35291	-83.44446	-3737.50122
2.1998	4.94208	-26.39032	-107.72330	-41.28551
2.2198	4.39501	-28.13872	-54.59007	3648.58924
2.2398	3.82822	-28.15404	60.67435	5903.03125
2.2598	3.28647	-25.55864	198.58765	5913.49609
2.2798	2.82327	-20.38629	310.47925	3623.41333

V. 結　　　言

正弦曲線を有するカム系の一般形の変位，加速度およびパルスを求める式を誘導した。そして(A)および(B)の二式を解きその値を比較し，また曲線を描いてみた。

何れも $t=0$ においては勿論，変位は零であるが加速度は両者ともその値は同じく $\ddot{y}=22,22232$ mm/sec² であるが $t=0.1$sec の場合(A)の場合，変位 $y=0.69622$mm に対し(B)の場合 $y=0.69235$ mm で，加速度は(A)の場合 $\ddot{y}=401,97778$mm/sec²，(B)の場合は$y=396,99878$mm/sec² でその値は出発点においては余り変化はない。

また(A)の場合 $t=0.4$sec の時 $\ddot{y}=-322,34570$mm/sec²

$t=0.6$sec の時 $\ddot{y}=-430,90454$mm/sec²

$t=1.1$sec の時 $\ddot{y}=403,29956$mm/sec²

(B)の場合 $t=0.4$sec の時 $\ddot{y}=-321,18359$mm/sec²

$t=0.6$sec の時 $\ddot{y}=-415,38867$mm/sec²

$t=1.1$sec の時 $\ddot{y}=381,20239$

となる。よってカム支持軸による速度項，すなわち軸の回転摩擦による減衰項は $0.003\dot{y}$ を $0.02\dot{y}$ に変えても振動系における加速力の大きさには余り影響はないが，スプリング系とカム回転力により甚しく影響することを知る。

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(著者機械工学科昭和48年1月10日受理)